5 Transformation between the International Terrestrial Reference System and the Geocentric Celestial Reference System (10 August 2012)

5.1 Introduction

The transformation to be used to relate the International Terrestrial Reference System (ITRS) to the Geocentric Celestial Reference System (GCRS) at the date t of the observation can be written as:

$$[\text{GCRS}] = Q(t)R(t)W(t) \, [\text{ITRS}], \tag{5.1}$$

where $Q(t)$, $R(t)$ and $W(t)$ are the transformation matrices arising from the motion of the celestial pole in the celestial reference system, from the rotation of the Earth around the axis associated with the pole, and from polar motion, respectively.

Note that Eq. (5.1) is valid for any choice of celestial pole and origin on the equator of that pole.

The definition of the GCRS and ITRS and the procedures for the ITRS to GCRS transformation that are provided in this chapter comply with the IAU 2000/2006 resolutions (provided at $<1>$ and in Appendix B). More detailed explanations about the relevant concepts, software and IERS products corresponding to the IAU 2000 resolutions can be found in IERS Technical Note 29 (Capitaine *et al.*, 2002), as well as in a number of original subsequent publications that are quoted in the following sections.

The chapter follows the recommendations on terminology associated with the IAU 2000/2006 resolutions that were made by the 2003-2006 IAU Working Group on "Nomenclature for fundamental astronomy" (NFA) (Capitaine *et al.*, 2007). We will refer to those recommendations in the following as "NFA recommendations" (see Appendix A for the list of the recommendations). This chapter also uses the definitions that were provided by this Working Group in the "IAU 2006 NFA Glossary" (available at
`http://syrte.obspm.fr/iauWGnfa/`).

Eq. (5.1), as well as the following formulas in this chapter, are theoretical formulations that refer to reference "systems". However, it should be clear that the numerical implementation of those formulas involves the IAU/IUGG adopted realization of those reference systems, *i.e.* the International Terrestrial Reference Frame (ITRF) and the International Celestial Reference Frame (ICRF), respectively.

Numerical values contained in this chapter have been revised from the IERS 2003 values in order to be compliant with the IAU 2006 precession. Software routines implementing the transformations are described towards the end of the chapter.

The transformation between the celestial and terrestrial reference systems also being required for computing directions of celestial objects in intermediate systems, the process to transform among these systems consistently with the IAU resolutions is also set out at the end of this chapter, including a chart provided by the IAU NFA Working Group in order to illustrate the various stages of the process.

5.2 The framework of IAU 2000/2006 resolutions

Several resolutions were adopted by the XXIVth and XXVIth General Assemblies of the International Astronomical Union (Manchester, August 2000, and Prague, August 2006) that concern the transformation between the celestial and terrestrial reference systems (see [1] and Appendix B for the complete text of those resolutions). Those resolutions were endorsed by the IUGG in 2003 and 2007, respectively.

5.2.1 IAU 2000 resolutions

The IAU 2000 resolutions ($<^1>$) that are relevant to this chapter are described in the following:

- IAU 2000 Resolution B1.3 specifies that the systems of space-time coordinates as defined by IAU 1991 Resolution A4 for the solar system and the Earth within the framework of General Relativity are now named the Barycentric Celestial Reference System (BCRS) and the Geocentric Celestial Reference System (GCRS), respectively. It also provides a general framework for expressing the metric tensor and defining coordinate transformations at the first post-Newtonian level.

- IAU 2000 Resolution B1.6 recommends that, beginning on 1 January 2003, the IAU 1976 precession model (Lieske *et al.*, 1977) and the IAU 1980 theory of nutation (Wahr, 1981; Seidelmann, 1982) be replaced by the precession-nutation model IAU 2000A (MHB2000 based on the transfer functions of Mathews *et al.* (2002)) for those who need a model at the 0.2 mas level, or its shorter version IAU 2000B for those who need a model only at the 1 mas level, together with their associated celestial pole offsets, published in this document. See Dehant *et al.* (1999) for more details. In addition to that model are frame bias values between the mean pole and equinox at J2000.0 and the GCRS.

  The precession part of the IAU 2000A model consists only of corrections to the precession rates of the IAU 1976 precession, and hence does not correspond to a dynamical theory. This is why IAU 2000 Resolution B1.6, that recommended the IAU 2000A precession-nutation model, encouraged at the same time the development of new expressions for precession consistent with dynamical theories and with IAU 2000A nutation.

- IAU 2000 Resolution B1.7 recommends that the Celestial Intermediate Pole (CIP) be implemented in place of the Celestial Ephemeris Pole (CEP) as of 1 January 2003, and specifies how to implement its definition through its direction at J2000.0 in the GCRS as well as the realization of its motion both in the GCRS and ITRS. Its definition is an extension of that of the CEP in the high frequency domain and coincides with that of the CEP in the low frequency domain (Capitaine, 2000).

- IAU 2000 Resolution B1.8 recommends the "non-rotating origin" (Guinot, 1979) for longitude origins on the CIP equator in the celestial and the terrestrial reference systems. They were designated *Celestial Ephemeris Origin* and *Terrestrial Ephemeris Origin*, but renamed *Celestial Intermediate Origin* and *Terrestrial Intermediate Origin*, respectively, by IAU 2006 Resolution B2 (see below). IAU 2000 Resolution B1.8 defines the Earth Rotation Angle (ERA) as the angle measured along the equator of the CIP between the CIO and the TIO, and defines UT1 by a conventionally adopted linear proportionality to the ERA. IAU 2000 Resolution B1.8 also recommends that the transformation between the ITRS and GCRS be specified by the position of the CIP in the GCRS, the position of the CIP in the ITRS, and the Earth Rotation Angle. It was finally recommended that the IERS take steps to implement this by 1 January 2003 and that the IERS continue to provide users with data and algorithms for the conventional transformation.

IAU 2000 Resolutions B1.6, B1.7 and B1.8 came into force on 1 January 2003. By that time, the required models, procedures, data and software had been made available by the IERS Conventions 2003 and the Standards Of Fundamental Astronomy (SOFA) service (Section 5.9 and Wallace, 1998) and the resolutions were implemented operationally.

5.2.2 IAU 2006 resolutions

The IAU 2006 resolutions (*cf.* Appendix B) that are relevant to this chapter are the following:

- IAU 2006 Resolution B1, proposed by the 2003-2006 IAU Working Group on "Precession and the Ecliptic" (Hilton *et al.*, 2006), recommends that, beginning on 1 January 2009, the precession component of the IAU 2000A precession-nutation model be replaced by the P03

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[1] http://syrte.obspm.fr/IAU_resolutions/Resol-UAI.htm

precession theory of Capitaine *et al.* (2003c) in order to be consistent with both dynamical theories and the IAU 2000 nutation. We will refer to that model as "IAU 2006 precession". That resolution also clarifies the definitions of the precession of the equator and the precession of the ecliptic.

- IAU 2006 Resolution B2, which is a supplement to the IAU 2000 resolutions on reference systems, consists of two recommendations:

  1. harmonizing "intermediate" in the names of the pole and the origin (*i.e.* celestial and terrestrial intermediate origins, CIO and TIO instead of CEO and TEO, respectively) and defining the celestial and terrestrial "intermediate" systems; and
  2. fixing the default orientation of the BCRS and GCRS, which are, unless otherwise stated, assumed to be oriented according to the ICRS axes.

The IAU precession-nutation model resulting from IAU 2000 Resolution B1.6 and IAU 2006 Resolution B1, will be denoted IAU 2006/2000 precession-nutation in the following. A description of that model is given in Section 5.6.1 and Section 5.6.2.

5.3 Implementation of IAU 2000 and IAU 2006 resolutions

5.3.1 The IAU 2000/2006 space-time reference systems

In order to follow IAU 2000 Resolution B1.3, the celestial reference system to be considered in the terrestrial-to-celestial transformation expressed by Eq. (5.1) must correspond to the geocentric space coordinates of the GCRS. IAU 1991 Resolution A4 specified that the relative orientation of barycentric and geocentric spatial axes in BCRS and GCRS are without any time-dependent rotation. This requires that the geodesic precession and nutation be taken into account in the precession-nutation model. Moreover, IAU 2006 Resolution B2 specifies that the BCRS and GCRS are oriented according to the ICRS axes.

Concerning the time coordinates, IAU 1991 Resolution A4 defined TCB and TCG of the BCRS and GCRS respectively, as well as another time coordinate in the GCRS, Terrestrial Time (TT), which is the theoretical counterpart of the realized time scale $TAI + 32.184$ s. The IAU 2000/2006 resolutions have clarified the definition of the two time scales TT and TDB. TT has been re-defined (IAU 2000 Resolution B1.9) as a time scale differing from TCG by a constant rate, which is a defining constant. In a very similar way, TDB has been re-defined (IAU 2006 Resolution B3) as a linear transformation of TCB, the coefficients of which are defining constants. See Chapter 10 for the relationships between these time scales.

The parameter t, used in Eq. (5.1) as well as in the following expressions, is defined by

$$t = (TT - 2000 \text{ January 1d 12h TT}) \text{ in days}/36525. \tag{5.2}$$

This definition is consistent with IAU 1994 Resolution C7 which recommends that the epoch J2000.0 be defined at the geocenter and at the date 2000 January 1.5 TT = Julian Date 2451545.0 TT.

5.3.2 Schematic representation of the motion of the Celestial Intermediate Pole (CIP)

According to IAU 2000 Resolution B1.7, the CIP is an intermediate pole separating, by convention, the motion of the pole of the ITRS in the GCRS into a celestial part and a terrestrial part. The convention is such that:

- the celestial motion of the CIP (precession-nutation), includes all the terms with periods greater than 2 days in the GCRS (*i.e.* frequencies between -0.5 cycles per sidereal day (cpsd) and $+0.5$ cpsd),
- the terrestrial motion of the CIP (polar motion), includes all the terms outside the retrograde diurnal band in the ITRS (*i.e.* frequencies lower than -1.5 cpsd or greater than -0.5 cpsd).

The following chart illustrates the conventional frequency separation between the precession-nutation of the CIP and its polar motion, either viewed in the ITRS (top), or the GCRS (bottom), with a 1 cpsd shift due to the rotation of the ITRS with respect to the GCRS.

frequency in ITRS $\quad$ _ _ _ _ | _____ | _____ | _____ | _____ | _____ | _____ | _ _ _ _
$$-3.5 \qquad -2.5 \qquad -1.5 \qquad -0.5 \qquad +0.5 \qquad +1.5 \qquad +2.5 \ \text{(cpsd)}$$
$\qquad\qquad$ _____ *polar motion* | $\qquad$ | *polar motion* _____

frequency in GCRS $\quad$ _ _ _ _ | _____ | _____ | _____ | _____ | _____ | _____ | _ _ _ _
$$-2.5 \qquad -1.5 \qquad -0.5 \qquad +0.5 \qquad +1.5 \qquad +2.5 \qquad +3.5 \ \text{(cpsd)}$$
$\qquad\qquad\qquad\qquad\qquad$ | *precession* |
$\qquad\qquad\qquad\qquad\qquad$ | *-nutation* |

5.3.3 The IAU 2000/2006 realization of the Celestial Intermediate Pole (CIP)

In order to follow IAU 2000 Resolution B1.6 and IAU 2006 Resolution B1, the precession-nutation quantities to be used in the transformation matrix $Q(t)$ of Eq. (5.1) must, starting on 1 January 2009, be based on the IAU 2006 precession and on the nutation model IAU 2000A or IAU 2000B depending on the required precision (the corresponding precession-nutation being denoted IAU 2006/2000A and IAU 2006/200B, respectively). IAU 2006 Resolution B1 adopting the IAU 2006 precession does not stipulate a specific parameterization, expressly stating that the user makes this choice. Various ways of forming the precession-nutation matrix based on a rigorous procedure in the IAU 2006 framework have been discussed in Capitaine and Wallace (2006), and the precession-nutation procedures consistent with IAU 2006 resolutions have been provided in Wallace and Capitaine (2006). The procedures provided in the following sections are based on the results of that paper.

In order to follow IAU 2000 Resolution B1.7, the realized celestial pole must be the CIP. This requires an offset at epoch in the conventional model for precession-nutation as well as diurnal and higher frequency variations in the Earth's orientation. According to this resolution, the direction of the CIP at J2000.0 has to be offset from the pole of the GCRS in a manner consistent with the IAU 2006/2000A precession-nutation model. The motion of the CIP in the GCRS is realized (see Section 5.3.2) by the IAU model for precession and forced nutation for periods greater than two days plus additional time-dependent corrections provided by the IERS through appropriate astro-geodetic observations. The motion of the CIP in the ITRS is provided by the IERS through astro-geodetic observations and models including variations with frequencies outside the retrograde diurnal band.

The realization of the CIP thus requires that the IERS monitor the observed differences (reported as "celestial pole offsets") with respect to the conventional celestial position of the CIP in the GCRS based on the IAU 2006/2000 precession-nutation model together with its observed offset at epoch. It also requires that the motion of the CIP in the ITRS be provided by the IERS by observations taking into account a predictable part specified by a model including the terrestrial motion of the pole corresponding to the forced nutations with periods less than two days (in the GCRS) as well as the tidal variations in polar motion.

5.3.4 Procedures to be used for the terrestrial-to-celestial transformation consistent with IAU 2000/2006 resolutions

Two equivalent procedures have been given in the IERS Conventions 1996 and 2003 for the coordinate transformation from the ITRS to the GCRS expressed by Eq. (5.1). According to the NFA recommendations, these procedures, which differ by the origin that is adopted on the CIP equator (*i.e.* the equinox or the CIO), will be called in the following "equinox based" and "CIO based", respectively. Each of these procedures is based on a specific representation of the transformation matrix components $Q(t)$ and $R(t)$ of Eq. (5.1), which depends on the corresponding origin on the CIP equator, while the representation of the transformation matrix component $W(t)$ is common to the two procedures.

IAU 2000 Resolutions B1.3, B1.6 and B1.7 as well as IAU 2006 B1 can be implemented in any of these two procedures if the requirements described in Sections 5.3.1 and 5.3.3 are followed for the space-time coordinates in the geocentric celestial reference system. However, only the CIO based procedure can be in agreement with IAU 2000 Resolution B1.8, which requires the use of the "non-rotating origin" in both the GCRS and the ITRS as well as the position of the CIP in the GCRS and in the ITRS. This is why this chapter of the IERS Conventions provides the expressions for the implementation of the IAU resolutions with more emphasis on the CIO based procedure. However, the IERS must also provide users with data and algorithms for the conventional transformation, which implies in particular that the expression of Greenwich Sidereal Time (GST) has to be consistent with the new procedure. Consequently, this chapter also provides the expressions which are necessary to be compatible with the resolutions when using the conventional transformation.

The following sections give the details of the CIO based procedure and the standard expressions necessary to obtain the numerical values of the relevant parameters for both procedures at the date of observation.

5.4 Coordinate transformation consistent with the IAU 2000/2006 resolutions

The coordinate transformation (5.1) from the ITRS to the GCRS corresponding to the procedure consistent with IAU 2000 Resolution B1.8 is expressed in terms of the three fundamental components $W(t)$, $R(t)$ and $Q(t)$, as described in the following.

According to IAU 2006 Resolution B2, the system at date t as realized from the ITRS by applying the transformation $W(t)$ in both procedures is the "Terrestrial Intermediate Reference System" (TIRS). It uses the CIP as its z-axis and the TIO as its x-axis.

The CIO based procedure realizes an intermediate celestial reference system at date t that uses the CIP as its z-axis and the CIO as its x-axis. According to IAU 2006 Resolution B2, it is called the "Celestial Intermediate Reference System" (CIRS). It uses the Earth Rotation Angle in the transformation matrix $R(t)$, and the two coordinates of the CIP in the GCRS (Capitaine, 1990) in the transformation matrix $Q(t)$.

The classical, or equinox based, procedure realizes an intermediate celestial reference system at date t that uses the CIP as its z-axis and the equinox as its x-axis. It is called the "true equinox and equator of date system". It uses apparent Greenwich Sidereal Time (GST) in the transformation matrix $R(t)$ and the classical precession and nutation parameters in the transformation matrix $Q(t)$.

Each of the transformation matrix components $W(t)$, $R(t)$ and $Q(t)$ of Eq. (5.1) is a series of rotations about the axes 1, 2 and 3 of the coordinate frame. In the following, R_1, R_2 and R_3 denote rotation matrices with positive angle about the axes 1, 2 and 3. The position of the CIP both in the ITRS and GCRS is provided by the x and y components of the CIP unit vector. These components are called "coordinates" in the following, and their numerical expressions are multiplied by the factor $1296000''/2\pi$ in order to represent the approximate values in arcseconds of the corresponding "angles" (strictly their sines) with respect to the z-axis of the reference system.

5.4.1 Expression for the transformation matrix for polar motion

The transformation matrix arising from polar motion (i.e. relating ITRS and TIRS) can be expressed as:

$$W(t) = R_3(-s') \cdot R_2(x_p) \cdot R_1(y_p), \tag{5.3}$$

x_p and y_p being the "polar coordinates" of the Celestial Intermediate Pole (CIP) in the ITRS and s' being a quantity, named "TIO locator", which provides the position of the TIO on the equator of the CIP corresponding to the kinematical definition of the "non-rotating" origin (NRO) in the ITRS when the CIP is moving with respect to the ITRS due to polar motion.

The expression of s' as a function of the coordinates x_p and y_p is:

$$s'(t) = \frac{1}{2} \int_{t_0}^{t} (x_p \dot{y}_p - \dot{x}_p y_p)\ dt. \tag{5.4}$$

The use of the quantity s', which was neglected in the classical form prior to 1 January 2003, is necessary to provide an exact realization of the "instantaneous prime meridian" (designated by "TIO meridian").

5.4.2 Expression for the CIO based transformation matrix for Earth rotation

The CIO based transformation matrix arising from the rotation of the Earth around the axis of the CIP (*i.e.* relating TIRS and CIRS), can be expressed as:

$$R(t) = R_3(-\text{ERA}), \tag{5.5}$$

where ERA is the Earth Rotation Angle between the CIO and the TIO at date t on the equator of the CIP, which is the rigorous definition of the sidereal rotation of the Earth.

5.4.3 Expression for the equinox based transformation matrix for Earth rotation

The equinox based transformation matrix $R(t)$ for Earth rotation transforms from the TIRS to the true equinox and equator of date system. It uses Greenwich (apparent) Sidereal Time, *i.e.* the angle between the equinox and the TIO, to represent ERA in Eq. (5.5).

5.4.4 Expression for the transformation matrix for the celestial motion of the CIP

The CIO based transformation matrix arising from the motion of the CIP in the GCRS (*i.e.* relating CIRS and GCRS), can be expressed as:

$$Q(t) = R_3(-E) \cdot R_2(-d) \cdot R_3(E) \cdot R_3(s), \tag{5.6}$$

E and d being such that the coordinates of the CIP in the GCRS are:

$$X = \sin d \cos E, \qquad Y = \sin d \sin E, \qquad Z = \cos d, \tag{5.7}$$

and s being a quantity, named "CIO locator", which provides the position of the CIO on the equator of the CIP corresponding to the kinematical definition of the NRO in the GCRS when the CIP is moving with respect to the GCRS, between the reference epoch and the date t due to precession and nutation. Its expression as a function of the coordinates X and Y is (Capitaine *et al.*, 2000)

$$s(t) = -\int_{t_0}^{t} \frac{X(t)\dot{Y}(t) - Y(t)\dot{X}(t)}{1 + Z(t)} dt - (\sigma_0 N_0 - \Sigma_0 N_0), \tag{5.8}$$

where σ_0 and Σ_0 are the positions of the CIO at J2000.0 and the x-origin of the GCRS respectively and N_0 is the ascending node of the equator at J2000.0 in the equator of the GCRS. Or equivalently, within 1 microarcsecond over one century:

$$s(t) = -\frac{1}{2}[X(t)Y(t) - X(t_0)Y(t_0)] + \int_{t_0}^{t} \dot{X}(t)Y(t)dt - (\sigma_0 N_0 - \Sigma_0 N_0). \tag{5.9}$$

The arbitrary constant $\sigma_0 N_0 - \Sigma_0 N_0$, which had been conventionally chosen to be zero in previous references (*e.g.* Capitaine *et al.*, 2000), was afterwards chosen to ensure continuity with the classical procedure on 1 January 2003 (see expression (5.31)).

$Q(t)$ can be given in an equivalent form directly involving X and Y as

$$Q(t) = \begin{pmatrix} 1 - aX^2 & -aXY & X \\ -aXY & 1 - aY^2 & Y \\ -X & -Y & 1 - a(X^2 + Y^2) \end{pmatrix} \cdot R_3(s), \tag{5.10}$$

with $a = 1/(1 + \cos d)$, which can also be written, with an accuracy of $1\,\mu$as, as $a = 1/2 + 1/8(X^2 + Y^2)$. Such an expression of the transformation (5.1) leads to very simple expressions of the partial derivatives of observables with respect to the terrestrial coordinates of the CIP, UT1, and celestial coordinates of the CIP.

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5.4.5 Expression for the equinox based transformation matrix for precession-nutation

The equinox based matrix $Q(t)$ that transforms from the true equinox and equator of date system to the GCRS can be expressed in several ways corresponding to different parameterization choices.

- One rigorous way is that recommended in the previous version of the IERS Conventions (2003). It is composed of the classical nutation matrix (using the nutation angles $\Delta\psi$, $\Delta\epsilon$ in longitude and obliquity referred to the ecliptic of date and the mean obliquity of date, ϵ_A), the precession matrix including four rotations $(R_1(-\epsilon_0) \cdot R_3(\psi_A) \cdot R_1(\omega_A) \cdot R_3(-\chi_A))$, and a separate rotation matrix for the frame biases. The precession angles are those of Lieske et al. (1977), in which ϵ_0 is the obliquity of the ecliptic at J2000.0, ψ_A and ω_A are the precession quantities in longitude and obliquity referred to the ecliptic of epoch and χ_A is the precession of the ecliptic along the equator.

- Another rigorous way is that proposed by Fukushima (2003) as an extension to the GCRS of the method originally proposed by Williams (1994). This way is more concise than the previous one, as it can be referred directly to the GCRS pole and origin without requiring the frame bias to be applied separately, and there is no need for separate precession and nutation steps. It is composed of the four rotations: $R_1(-\epsilon) \cdot R_3(-\psi) \cdot R_1(\bar{\phi}) \cdot R_3(\bar{\gamma}))$, where the angles ϵ and ψ are each obtained by summing the contributions from the bias, precession and nutation in obliquity and ecliptic longitude, respectively, $\bar{\phi}$ is the obliquity of the ecliptic of date on the GCRS equator, and $\bar{\gamma}$ is the GCRS right ascension of the intersection of the ecliptic of date with the GCRS equator. Refer to Eq. (5.40) for derived expressions.

5.5 Parameters to be used in the transformation

5.5.1 Motion of the Celestial Intermediate Pole in the ITRS

The standard pole coordinates to be used for the parameters x_p and y_p appearing in Eq. (5.3) of Section 5.4.1, if not estimated from the observations, are those published by the IERS with additional components to account for the effect of ocean tides $(\Delta x, \Delta y)_{\text{ocean tides}}$ and for forced terms $(\Delta x, \Delta y)_{\text{libration}}$ with periods less than two days in space:

$$(x_p, y_p) = (x, y)_{\text{IERS}} + (\Delta x, \Delta y)_{\text{ocean tides}} + (\Delta x, \Delta y)_{\text{libration}}, \tag{5.11}$$

where $(x, y)_{\text{IERS}}$ are pole coordinates provided by the IERS, $(\Delta x, \Delta y)_{\text{ocean tides}}$ are the diurnal and semi-diurnal variations in pole coordinates caused by ocean tides, and $(\Delta x, \Delta y)_{\text{libration}}$ are the variations in pole coordinates corresponding to motions with periods less than two days in space that are not part of the IAU 2000 nutation model. These variations are described in detail below.

5.5.1.1 *Account of ocean tidal and libration effects in pole coordinates x_p and y_p*

The subdaily variations are not part of the polar motion values reported to and distributed by the IERS and are therefore to be added after interpolation. This is appropriately done by the routine "INTERP.F" of the IERS EOP Product Center, which interpolates series of x_{IERS}, y_{IERS} values to a chosen date and then adds the contribution for this date of (i) the tidal terms $(\Delta x, \Delta y)_{\text{ocean tides}}$ derived from Tables 8.2a and 8.2b, and (ii) the diurnal components of the $(\Delta x, \Delta y)_{\text{libration}}$ terms, derived from Table 5.1a. The long-period terms, as well as the secular variation of the libration contribution, are already contained in the observed polar motion and need not be added to the reported values $(x, y)_{\text{IERS}}$.

5.5.1.2 *Variations $(\Delta x, \Delta y)_{\text{ocean tides}}$ in polar motion*

These are tidal variations in Earth orientation considered in Chapter 8, including diurnal and semi-diurnal variations in pole coordinates caused by ocean tides. Tables 8.2a and 8.2b provide the amplitudes and arguments for the 71 tidal constituents of those diurnal and semi-diurnal variations that have been derived from the routine "ORTHO_EOP.F" based on the model from Ray *et al.* (1994). That routine is available on the website of the IERS Conventions (see Chapter 8).

5.5.1.3 *Variations* $(\Delta x, \Delta y)_{\text{libration}}$ *in polar motion*

According to the definition of the CIP (IAU 2000 Resolution B1.7; see Appendix A7), forced motions with periods less than two days in space are not included in the IAU 2000 nutation model and therefore have to be considered using a model for the corresponding motion of the pole in the ITRS.

Recent models for rigid Earth nutation (Bretagnon *et al.*, 1997; Folgueira *et al.*, 1998a and b; Souchay *et al.*, 1999; Roosbeek, 1999; Bizouard *et al.*, 2000 and 2001) include prograde diurnal and prograde semi-diurnal terms with respect to the GCRS with amplitudes up to $\sim 15\ \mu$as. The semi-diurnal terms in nutation have also been provided both for rigid and nonrigid Earth models based on Hamiltonian formalism (Getino *et al.*, 2001; Escapa *et al.*, 2002 and 2003).

These diurnal and semi-diurnal nutations, which, according to Chao *et al.* (1991), are designated here as "libration", originate from the direct effect of the external (mainly luni-solar) torque on the non-axisymmetric part of the Earth as expressed by the non-zonal terms of the geopotential. This effect is also called the "tidal gravitation" effect, and has been designated in the past as the "nutation" effect on polar motion.

The prograde diurnal nutations correspond to prograde and retrograde long-period variations in polar motion including the linear term, and the prograde semi-diurnal nutations correspond to prograde diurnal variations in polar motion (see for example Folgueira *et al.*, 2001). A table for operational use of the model for these variations in polar motion for a nonrigid Earth has been provided by an *ad hoc* Working Group (Brzeziński, 2002; Brzeziński and Mathews, 2003), based on nonrigid Earth models and developments of the tide generating potential (TGP; Brzeziński, 2001; Brzeziński and Capitaine, 2003; Mathews and Bretagnon, 2003). All components with amplitudes greater than $0.5\ \mu$as are given in Table 5.1a. The amplitudes of the diurnal terms are in very good agreement with those estimated by Getino *et al.* (2001). The contribution from the triaxiality of the core to the diurnal terms, while it can exceed the adopted cut-off level (Escapa *et al.*, 2002; Mathews and Bretagnon, 2003), has not been taken into account in the table due to the large uncertainty in the triaxiality of the core (Brzeziński and Capitaine, 2003; Dehant, 2002, private communication). The coefficients of Table 5.1a are based on Stokes coefficients of the JGM-3 geopotential model (Tapley *et al.*, 1996), but any of the geopotential models commonly used in current precision orbital analysis would give values that would agree within the adopted cut-off level of $0.1\ \mu$as with those of Table 5.1a.

The diurnal components of $(\Delta x, \Delta y)_{\text{libration}}$ can be computed with the routine PMSDNUT2.F, available on the Conventions Center website at $<^2>$. These diurnal components (namely, the 10 terms listed in Table 5.1a with periods near 1 day) should be considered similarly to the diurnal and semi-diurnal variations due to ocean tides (see above).

Note that the 10 diurnal terms of Table 5.1a due to the influence of tidal gravitation on the non-axisymmetric part of the Earth are a subset of the 41 diurnal terms of Table 8.2a with, for each common term, an amplitude about 10 times lower than the corresponding amplitude for the variation in UT1 (or LOD) caused by ocean tides provided in Table 8.2a.

5.5.2 Position of the Terrestrial Intermediate Origin in the ITRS

The quantity s' (*i.e.* the TIO locator) appearing in Eq. (5.3) and expressed by Eq. (5.4) is sensitive only to the largest variations in polar motion. Some components of s' have to be evaluated, in principle, from the measurements and can be extrapolated using the IERS data. Its main component can be written as:

$$s' = -0.0015 \left(a_c^2/1.2 + a_a^2 \right) t, \tag{5.12}$$

a_c and a_a being the average amplitudes (in as) of the Chandlerian and annual wobbles, respectively, in the period considered (Capitaine *et al.*, 1986). The value of s' will therefore be less than 0.4 mas after one century, even if the amplitudes for the Chandlerian and annual wobbles reach values

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^{2}`ftp://tai.bipm.org/iers/conv2010/chapter5/`

Table 5.1a: Coefficients of sin(argument) and cos(argument) in $(\Delta x, \Delta y)_{\text{libration}}$ due to tidal gravitation (of degree n) for a nonrigid Earth. Listed are all terms with amplitudes greater than 0.5 muas. Units are μas, γ denotes GMST+π (where GMST = ERA + precession in RA (see Eq. (5.32))). The expressions for the fundamental arguments (Delaunay arguments) are given by Eq. (5.43).

		Argument						Doodson	Period	x_p		y_p	
n	Tide	γ	l	l'	F	D	Ω	number	(days)	sin	cos	sin	cos
4		0	0	0	0	0	−1	055.565	6798.3837	0.0	0.6	−0.1	−0.1
3		0	−1	0	1	0	2	055.645	6159.1355	1.5	0.0	−0.2	0.1
3		0	−1	0	1	0	1	055.655	3231.4956	−28.5	−0.2	3.4	−3.9
3		0	−1	0	1	0	0	055.665	2190.3501	−4.7	−0.1	0.6	−0.9
3		0	1	1	−1	0	0	056.444	438.35990	−0.7	0.2	−0.2	−0.7
3		0	1	1	−1	0	−1	056.454	411.80661	1.0	0.3	−0.3	1.0
3		0	0	0	1	−1	1	056.555	365.24219	1.2	0.2	−0.2	1.4
3		0	1	0	1	−2	1	057.455	193.55971	1.3	0.4	−0.2	2.9
3		0	0	0	1	0	2	065.545	27.431826	−0.1	−0.2	0.0	−1.7
3		0	0	0	1	0	1	065.555	27.321582	0.9	4.0	−0.1	32.4
3		0	0	0	1	0	0	065.565	27.212221	0.1	0.6	0.0	5.1
3		0	−1	0	1	2	1	073.655	14.698136	0.0	0.1	0.0	0.6
3		0	1	0	1	0	1	075.455	13.718786	−0.1	0.3	0.0	2.7
3		0	0	0	3	0	3	085.555	9.1071941	−0.1	0.1	0.0	0.9
3		0	0	0	3	0	2	085.565	9.0950103	−0.1	0.1	0.0	0.6
2	Q_1'	1	−1	0	−2	0	−1	135.645	1.1196992	−0.4	0.3	−0.3	−0.4
2	Q_1	1	−1	0	−2	0	−2	135.655	1.1195149	−2.3	1.3	−1.3	−2.3
2	ρ_1	1	1	0	−2	−2	−2	137.455	1.1134606	−0.4	0.3	−0.3	−0.4
2	O_1'	1	0	0	−2	0	−1	145.545	1.0759762	−2.1	1.2	−1.2	−2.1
2	O_1	1	0	0	−2	0	−2	145.555	1.0758059	−11.4	6.5	−6.5	−11.4
2	M_1	1	−1	0	0	0	0	155.655	1.0347187	0.8	−0.5	0.5	0.8
2	P_1	1	0	0	−2	2	−2	163.555	1.0027454	−4.8	2.7	−2.7	−4.8
2	K_1	1	0	0	0	0	0	165.555	0.9972696	14.3	−8.2	8.2	14.3
2	K_1'	1	0	0	0	0	−1	165.565	0.9971233	1.9	−1.1	1.1	1.9
2	J_1	1	1	0	0	0	0	175.455	0.9624365	0.8	−0.4	0.4	0.8
Rate of secular polar motion (μas/y) due to the zero frequency tide													
4		0	0	0	0	0	0	555.555		−3.8		−4.3	

of the order of 0.5″ and 0.1″, respectively. Using the current mean amplitudes for the Chandlerian and annual wobbles gives (Lambert and Bizouard, 2002):

$$s' = -47 \ \mu\text{as} \ t. \tag{5.13}$$

5.5.3 Earth Rotation Angle

The conventional relationship defining UT1 from the Earth Rotation Angle (ERA) to be used in Eq. (5.5) of Section 5.4.2 is that given by Capitaine $et\ al.$ (2000):

$$\text{ERA}(T_u) = 2\pi(0.7790572732640 + 1.00273781191135448T_u), \tag{5.14}$$

where T_u=(Julian UT1 date−2451545.0), and UT1=UTC+(UT1−UTC), or equivalently (modulo 2π), in order to reduce possible rounding errors,

$$\begin{aligned} \text{ERA}(T_u) \ = \ & 2\pi(\text{UT1 Julian day fraction} \\ & +0.7790572732640 + 0.00273781191135448T_u). \end{aligned} \tag{5.15}$$

This definition of UT1 based on the CIO is insensitive at the microarcsecond level to the precession-nutation model and to the observed celestial pole offsets. Therefore, in processing observational data, the quantity s provided by Table 5.2d must be considered as independent of observations.

The above relationship also provides the ERA corresponding to a given UT1, the quantity UT1−UTC to be used (if not estimated from the observations) being the IERS value. Note that, for 0^h UT1, the UT1 Julian day fraction in Eq. (5.15) is 0.5.

Similarly to polar motion (cf. Section 5.5.1), additional components should be added to the values published by the IERS for UT1 and LOD to account for the effects of ocean tides and libration. These effects are described in detail below.

5.5.3.1 *Account of ocean tidal and libration effects in UT1 and LOD*

The subdaily variations are not part of the UT1 or LOD values reported to and distributed by the IERS and are therefore to be added after interpolation. This is appropriately done for the first effect by the routine "INTERP.F" of the IERS EOP Product Center, which interpolates series of $UT1_{IERS}$ and LOD_{IERS} values to a chosen date and then adds the contribution for this date of the tidal terms $\Delta UT1_{ocean\,tides}$, or $\Delta LOD_{ocean\,tides}$, derived from Tables 8.3a and 8.3b. The semi-diurnal components of the libration contribution $\Delta UT1_{libration}$, or $\Delta LOD_{libration}$ derived from Table 5.1b should be included in that routine.

5.5.3.2 *Variations $\Delta UT1_{ocean\,tides}$ and $\Delta LOD_{ocean\,tides}$ in UT1 and LOD*

These are tidal variations in Earth orientation considered in Chapter 8, including diurnal and semi-diurnal variations in UT1 or LOD caused by ocean tides. Tables 8.3a and 8.3b provide the amplitudes and arguments for the 71 tidal constituents of those diurnal and semi-diurnal variations that have been derived from the routine "ORTHO_EOP.F" (available on the website of the IERS Conventions at $<$[3]$>$) based on the model from Ray *et al.* (1994).

5.5.3.3 *Variations $\Delta UT1_{libration}$ and $\Delta LOD_{libration}$ in UT1 and LOD*

The axial component of Earth rotation contains small variations due to the direct effect of the external (mainly luni-solar) torque on the non-axisymmetric part of the Earth as expressed by the non-zonal terms of the geopotential. This effect was theoretically predicted by Tisserand (1891) and Kinoshita (1977), but was neglected in current practical applications due to its very small size (maximum peak-to-peak variation of about 0.1 mas, *i.e.* 0.007 ms in UT1). More complete descriptions were published when the effect, also designated as "tidal gravitation", became observationally detectable by, *e.g.* Chao *et al.* (1991), Wünsch (1991), Chao *et al.* (1996) and Brzeziński and Capitaine (2003; 2010).

An analytical solution for the sub-diurnal libration in UT1 has been derived by Brzeziński and Capitaine (2003) for the structural model of the Earth consisting of an elastic mantle and a liquid core which are not coupled to each other. The reference solution for the rigid Earth has been computed by using the satellite-determined coefficients of geopotential and the recent developments of the tide generating potential. Table 5.1b provides the amplitudes and arguments for all components in UT1 and LOD with amplitudes greater than $0.5\,\mu as$ (*i.e.* $0.033\,\mu s$ in UT1.) It consists of 11 semi-diurnal harmonics due to the influence of the TGP term u_{22} on the equatorial flattening of the Earth expressed by the Stokes coefficients C_{22}, S_{22}. There is excellent agreement between the values for the rigid Earth and the amplitudes derived by Wünsch (1991), except for the term with the tidal code ν_2, which seems to have been overlooked in the latter model. The amplitudes computed for an elastic Earth with liquid core are in reasonable agreement with those derived by Chao et al. (1991), but the latter model was not complete.

Note that the 11 semi-diurnal terms of Table 5.1b due to the influence of tidal gravitation on the triaxiality of the Earth are a subset of the 30 semi-diurnal terms of Table 8.3b, with, for each common term, an amplitude about 10 times lower than the corresponding amplitude for the variation in pole coordinates caused by ocean tides provided in Table 8.3b. Nevertheless, the maximum peak-to-peak size of the triaxiality effect on UT1 is about 0.105 mas, hence definitely above the current uncertainty of UT1 determinations. A comparison with the corresponding model of prograde diurnal polar motion associated with the Earth's libration (Table 5.1a) shows that the two effects are of similar size and that there is consistency between the underlying dynamical models and the parameters employed. The sub-diurnal libration in

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[3]`ftp://tai.bipm.org/iers/conv2010/chapter8/`

Table 5.1b: Coefficients of sin(argument) and cos(argument) of semi-diurnal variations in UT1 and LOD due to libration for a non-rigid Earth. Listed are all terms with amplitudes of UT1 greater than 0.033 μs. Units are μs, γ denotes GMST+π. Expressions for the fundamental arguments are given by Eq. (5.43).

Tide	Argument						Doodson number	Period (days)	UT1		LOD	
	γ	l	l'	F	D	Ω			sin	cos	sin	cos
$2N_2$	2	-2	0	-2	0	-2	235.755	0.5377239	0.05	-0.03	-0.3	-0.6
μ_2	2	0	0	-2	-2	-2	237.555	0.5363232	0.06	-0.03	-0.4	-0.7
N_2	2	-1	0	-2	0	-2	245.655	0.5274312	0.35	-0.20	-2.4	-4.2
ν_2	2	1	0	-2	-2	-2	247.455	0.5260835	0.07	-0.04	-0.5	-0.8
M'_2	2	0	0	-2	0	-1	255.545	0.5175645	-0.07	0.04	0.5	0.8
M_2	2	0	0	-2	0	-2	255.555	0.5175251	1.75	-1.01	-12.2	-21.3
L_2	2	1	0	-2	0	-2	265.455	0.5079842	-0.05	0.03	0.3	0.6
T_2	2	0	-1	-2	2	-2	272.556	0.5006854	0.05	-0.03	-0.3	-0.6
S_2	2	0	0	-2	2	-2	273.555	0.5000000	0.76	-0.44	-5.5	-9.5
K_2	2	0	0	0	0	0	275.555	0.4986348	0.21	-0.12	-1.5	-2.6
K'_2	2	0	0	0	0	-1	275.565	0.4985982	0.06	-0.04	-0.4	-0.8

UT1, $\Delta UT1_{\text{libration}}$, can be computed with the routine UTLIBR.F, provided by A. Brzeziński, available on the Conventions Center website at $<2>$.

5.5.4   Forced motion of the Celestial Intermediate Pole in the GCRS

The coordinates of the CIP in the GCRS to be used for the parameters X and Y appearing in Eq. (5.10) of Section 5.4.4 can be given by developments as function of time of those quantities. Developments valid at the microarcsecond level, based on the IAU 2006 precession and IAU 2000A nutation (see Section 5.6 for more details) and on their corresponding pole and equinox offsets at J2000.0 with respect to the pole of the GCRS have been computed (Capitaine and Wallace, 2006). They replace the previous developments based on the IAU 2000 model for precession-nutation and frame biases that had been provided by Capitaine $et~al.$ (2003a) and in the IERS Conventions (2003).

The IAU 2006/2000A developments are as follows:

$$
\begin{aligned}
X = \quad & -0.016617'' + 2004.191898''t - 0.4297829''t^2 \\
& -0.19861834''t^3 + 0.000007578''t^4 + 0.0000059285''t^5 \\
& + \sum_i [(a_{s,0})_i \sin(\text{ARGUMENT}) + (a_{c,0})_i \cos(\text{ARGUMENT})] \\
& + \sum_i [(a_{s,1})_i t \sin(\text{ARGUMENT}) + (a_{c,1})_i t \cos(\text{ARGUMENT})] \\
& + \sum_i [(a_{s,2})_i t^2 \sin(\text{ARGUMENT}) + (a_{c,2})_i t^2 \cos(\text{ARGUMENT})] \\
& + \cdots ,
\end{aligned}
$$

$$
\begin{aligned}
Y = \quad & -0.006951'' - 0.025896''t - 22.4072747''t^2 \\
& +0.00190059''t^3 + 0.001112526''t^4 + 0.0000001358''t^5 \\
& + \sum_i [(b_{c,0})_i \cos(\text{ARGUMENT}) + (b_{s,0})_i \sin(\text{ARGUMENT})] \\
& + \sum_i [(b_{c,1})_i t \cos(\text{ARGUMENT}) + (b_{s,1})_i~ t \sin(\text{ARGUMENT})] \\
& + \sum_i [(b_{c,2})_i t^2 \cos(\text{ARGUMENT}) + (b_{s,2})_i t^2 \sin(\text{ARGUMENT})] \\
& + \cdots ,
\end{aligned}
\tag{5.16}
$$

the parameter t being given by expression (5.2) and ARGUMENT being a function of the fundamental arguments of the nutation theory, whose expressions are given by Eq. (5.43) for the lunisolar ones

and Eq. (5.44) for the planetary ones. The full IAU 2000/2006 series are available electronically on the IERS Conventions Center website (Tables 5.2a and 5.2b) at $<^2>$, `tab5.2a.txt` for the X coordinate and `tab5.2b.txt` for the Y coordinate. The polynomial terms of X and Y are given in (5.16). An extract from Tables 5.2a and 5.2b for the largest non-polynomial terms in X and Y is given below.

Table 5.2a: Extract from Table 5.2a (available at $<^2>$) for the largest non-polynomial terms (*i.e.* coefficients of the Fourier and Poisson terms, with t in Julian centuries) in the development (5.16) for $X(t)$ compatible with the IAU 2006/2000A precession-nutation model (unit μas). The expressions for the fundamental arguments appearing in columns 4 to 17 are given by Eq. (5.43) and Eq. (5.44). (Because the largest terms are all luni-solar, columns 9-17 contain only zeros in the extract shown.)

i	$(a_{s,0})_i$	$(a_{c,0})_i$	l	l'	F	D	Ω	L_{Me}	L_{Ve}	L_E	L_{Ma}	L_J	L_{Sa}	L_U	L_{Ne}	p_A
1	−6844318.44	1328.67	0	0	0	0	1	0	0	0	0	0	0	0	0	0
2	−523908.04	−544.75	0	0	2	−2	2	0	0	0	0	0	0	0	0	0
3	−90552.22	111.23	0	0	2	0	2	0	0	0	0	0	0	0	0	0
4	82168.76	−27.64	0	0	0	0	2	0	0	0	0	0	0	0	0	0
5	58707.02	470.05	0	1	0	0	0	0	0	0	0	0	0	0	0	0
.....																
i	$(a_{s,1})_i$	$(a_{c,1})_i$	l	l'	F	D	Ω	L_{Me}	L_{Ve}	L_E	L_{Ma}	L_J	L_{Sa}	L_U	L_{Ne}	p_A
1307	−3309.73	205833.11	0	0	0	0	1	0	0	0	0	0	0	0	0	0
1308	198.97	12814.01	0	0	2	−2	2	0	0	0	0	0	0	0	0	0
1309	41.44	2187.91	0	0	2	0	2	0	0	0	0	0	0	0	0	0
.....																

Table 5.2b: Extract from Table 5.2b (available at $<^2>$) for the largest non-polynomial terms (*i.e.* coefficients of the Fourier and Poisson terms, with t in Julian centuries) in the development (5.16) $Y(t)$ compatible with the IAU 2006/2000A precession-nutation model (unit μas). The expressions for the fundamental arguments appearing in columns 4 to 17 are given by Eq. (5.43) and Eq. (5.44). (Because the largest terms are all luni-solar, columns 9-17 contain only zeros in the extract shown.)

i	$(b_{s,0})_i$	$(b_{c,0})_i$	l	l'	F	D	Ω	L_{Me}	L_{Ve}	L_E	L_{Ma}	L_J	L_{Sa}	L_U	L_{Ne}	p_A
1	1538.18	9205236.26	0	0	0	0	1	0	0	0	0	0	0	0	0	0
2	−458.66	573033.42	0	0	2	−2	2	0	0	0	0	0	0	0	0	0
3	137.41	97846.69	0	0	2	0	2	0	0	0	0	0	0	0	0	0
4	−29.05	−89618.24	0	0	0	0	2	0	0	0	0	0	0	0	0	0
5	−17.40	22438.42	0	1	2	−2	2	0	0	0	0	0	0	0	0	0
.....																
i	$(b_{s,1})_i$	$(b_{c,1})_i$	l	l'	F	D	Ω	L_{Me}	L_{Ve}	L_E	L_{Ma}	L_J	L_{Sa}	L_U	L_{Ne}	p_A
963	153041.79	853.32	0	0	0	0	1	0	0	0	0	0	0	0	0	0
964	11714.49	−290.91	0	0	2	−2	2	0	0	0	0	0	0	0	0	0
965	2024.68	−51.26	0	0	2	0	2	0	0	0	0	0	0	0	0	0
.....																

The numerical values of the coefficients of the polynomial part of X and Y (*cf.* (5.16)) are derived from the development as a function of time of the precession in longitude and obliquity and pole offset at J2000.0 and the amplitudes $(a_{s,j})_i$, $(a_{c,j})_i$, $(b_{c,j})_i$, $(b_{s,j})_i$ for $j = 0, 1, 2, \ldots$ are derived from the amplitudes of the precession and nutation series. The amplitudes $(a_{s,0})_i$, $(b_{c,0})_i$ of the sine and cosine terms in X and Y respectively are equal to the amplitudes $A_i \times \sin \epsilon_0$ and B_i of the series for nutation in longitude [$\times \sin \epsilon_0$] and obliquity, except for a few terms in each coordinate X and Y which contain a contribution from crossed-nutation effects. The coordinates X and Y contain Poisson terms in $t\sin$, $t\cos$, $t^2\sin$, $t^2\cos$, ... which originate from crossed terms between precession and nutation.

The contributions (in μas) to expression (5.16) from the frame biases are

$$
\begin{aligned}
dX &= -16617.0 - 1.6\ t^2 + 0.7\ \cos\Omega, \\
dY &= -6819.2 - 141.9\ t + 0.5\ \sin\Omega,
\end{aligned}
\tag{5.17}
$$

the first term in each coordinate being the contribution from the celestial pole offset at J2000.0 and the following ones from the equinox offset at J2000.0, also called "frame bias in right ascension."

The polynomial changes in X and Y due to the change in the precession model from IAU 2000 to IAU 2006 can be written in μas as (Capitaine and Wallace, 2006, after correcting a typographical error in the t^2 term of dY):

$$
\begin{aligned}
dX &= 155\ t - 2564\ t^2 + 2\ t^3 + 54\ t^4, \\
dY &= -514\ t - 24\ t^2 + 58\ t^3 - 1\ t^4 - 1\ t^5.
\end{aligned}
\tag{5.18}
$$

In addition, there are slight changes in a few periodic terms of the IAU 2006 version of the X, Y series with respect to the IAU 2000 version, corresponding to additional Poisson terms in nutation caused by introducing the IAU 2006 J_2 rate value (see Section 5.6.3). The largest changes (Capitaine and Wallace, 2006) in μas are:

$$
\begin{aligned}
dX_{J2d} &= 18.8\ t \sin\Omega + 1.4\ t \sin 2(F - D + \Omega), \\
dY_{J2d} &= -25.6\ t \cos\Omega - 1.6\ t \cos 2(F - D + \Omega).
\end{aligned}
\tag{5.19}
$$

The periodic terms expressed by (5.19) are included in the IAU 2006/2000A version of the X, Y series. The difference between the IAU 2006 and IAU 2000A expressions for X and Y is available electronically on the IERS Conventions Center website (Table 5.2f) at $<^4>$ in file `tab5.2f.txt`.

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The relationships between the coordinates X and Y and the precession-nutation quantities are (Capitaine, 1990):

$$
\begin{aligned}
X &= \bar{X} + \xi_0 - d\alpha_0\ \bar{Y}, \\
Y &= \bar{Y} + \eta_0 + d\alpha_0\ \bar{X},
\end{aligned}
\tag{5.20}
$$

where ξ_0 and η_0 are the celestial pole offsets at the basic epoch J2000.0 and $d\alpha_0$ the right ascension of the mean equinox of J2000.0 in the GCRS (*i.e.* frame bias in right ascension). (See the numbers provided below in (5.21) and (5.33) for these quantities.)

The mean equinox of J2000.0 to be considered is not the "rotational dynamical mean equinox of J2000.0" as used in the past, but the "inertial dynamical mean equinox of J2000.0" to which the recent numerical or analytical solutions refer. The latter is associated with the ecliptic in the inertial sense, which is the plane perpendicular to the angular momentum vector of the orbital motion of the Earth-Moon barycenter as computed from the velocity of the barycenter relative to an *inertial system*. The rotational equinox is associated with the ecliptic in the rotational sense, which is perpendicular to the angular momentum vector computed from the velocity referred to the *rotating* orbital plane of the Earth-Moon barycenter. (The difference between the two angular momenta is the angular momentum associated with the rotation of the orbital plane.) See Standish (1981) for more details. The numerical value for $d\alpha_0$ as derived from Chapront *et al.* (2002) to be used in expression (5.20) is:

$$
d\alpha_0 = (-0.01460 \pm 0.00050)''.
\tag{5.21}
$$

Quantities $\bar{X}$ and $\bar{Y}$ are given by:

$$
\begin{aligned}
\bar{X} &= \sin\omega \sin\psi, \\
\bar{Y} &= -\sin\epsilon_0 \cos\omega + \cos\epsilon_0 \sin\omega \cos\psi
\end{aligned}
\tag{5.22}
$$

where ϵ_0 ($= 84381.406''$) is the IAU 2006 value for the obliquity of the ecliptic at J2000.0 (from Chapront *et al.*, 2002), ω, and ψ is the longitude, on the ecliptic of epoch, of the node of the true equator of date on the fixed ecliptic of epoch; these quantities are such that

$$
\omega = \omega_A + \Delta\epsilon_1; \qquad \psi = \psi_A + \Delta\psi_1,
\tag{5.23}
$$

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[4] `ftp://tai.bipm.org/iers/convupdt/chapter5/`

where ψ_A and ω_A are the precession quantities in longitude and obliquity (Lieske *et al.*, 1977) referred to the ecliptic of epoch and $\Delta\psi_1$, $\Delta\epsilon_1$ are the nutation angles in longitude and obliquity referred to the ecliptic of epoch. (See the numerical developments provided for the precession quantities in (5.39).) $\Delta\psi_1$, $\Delta\epsilon_1$ can be obtained from the nutation angles $\Delta\psi$, $\Delta\epsilon$ in longitude and obliquity referred to the ecliptic of date. The following formulation from Aoki and Kinoshita (1983) is accurate to better than one microarcsecond after one century:

$$
\begin{aligned}
\Delta\psi_1 \sin\omega_A &= \Delta\psi \sin\epsilon_A \cos\chi_A - \Delta\epsilon \sin\chi_A, \\
\Delta\epsilon_1 &= \Delta\psi \sin\epsilon_A \sin\chi_A + \Delta\epsilon \cos\chi_A,
\end{aligned}
\tag{5.24}
$$

ϵ_A being the mean obliquity of date and χ_A the precession of the ecliptic along the equator (Lieske *et al.*, 1977).

As VLBI observations have shown that there are deficiencies in the IAU 2006/2000A precession-nutation model of the order of 0.2 mas, mainly due to the fact that the free core nutation (FCN) (see Section 5.5.5) is not part of the model, the IERS will continue to publish observed estimates of the corrections to the IAU precession-nutation model. The observed differences with respect to the conventional celestial pole position defined by the models are monitored and reported by the IERS as "celestial pole offsets." Such time-dependent offsets from the direction of the pole of the GCRS must be provided as corrections δX and δY to the X and Y coordinates. These corrections can be related to the equinox based celestial pole offsets $\delta\psi$ (along the ecliptic of date) and $\delta\epsilon$ (in the obliquity of date) using the relationships (5.22) between X and Y and the precession-nutation quantities and (5.24) for the transformation from ecliptic of date to ecliptic of epoch. The relationship (5.25) below, which is to first order in the quantities, ensures an accuracy of one microarcsecond for one century, for values of $\delta\psi$ and $\delta\epsilon$ lower than 1 mas:

$$
\begin{aligned}
\delta X &= \delta\psi \sin\epsilon_A + (\psi_A \cos\epsilon_0 - \chi_A)\delta\epsilon, \\
\delta Y &= \delta\epsilon - (\psi_A \cos\epsilon_0 - \chi_A)\delta\psi \sin\epsilon_A.
\end{aligned}
\tag{5.25}
$$

These observed offsets include the contribution of the FCN described in Section 5.5.5. Using these offsets, the corrected celestial position of the CIP is given by

$$
X = X(\text{IAU } 2006/2000) + \delta X, \qquad Y = Y(\text{IAU } 2006/2000) + \delta Y.
\tag{5.26}
$$

This is practically equivalent to replacing the transformation matrix Q with the rotation

$$
\tilde{Q} = \begin{pmatrix} 1 & 0 & \delta X \\ 0 & 1 & \delta Y \\ -\delta X & -\delta Y & 1 \end{pmatrix} Q_{\text{IAU}},
\tag{5.27}
$$

where Q_{IAU} represents the $Q(t)$ matrix based on the IAU 2006/2000 precession-nutation model.

5.5.5 Free Core Nutation

Free core nutation is a free retrograde diurnal motion of the Earth's rotation axis with respect to the Earth caused by the interaction of the mantle and the fluid, ellipsoidal core as it rotates. Due to the definition of the CIP, this free motion appears as a motion of the CIP in the GCRS. Because this effect is a free motion with time-varying excitation and damping resulting in a variable amplitude and phase, a FCN model was not included in the IAU 2000A nutation model. As a result, a quasi-periodic un-modeled motion of the CIP in the GCRS at the 0.1–0.3 mas level still exists after the IAU 2006/2000A model has been taken into account.

Depending on accuracy requirements, the lack of a FCN model should cause negligible problems. However, for the most stringent accuracy applications, a FCN model may be incorporated to account for the FCN contribution to the CIP motion in the GCRS.

The FCN model of Lambert (2007) can be found at $<^5>$ and a copy is kept at the IERS Conventions Center website $<^2>$. It is expected that the coefficients of the model will be updated regularly

---

[5] http://syrte.obspm.fr/~lambert/fcn/

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Table 5.2c: Table for the coefficients of the empirical model of the retrograde FCN during the interval 1984-2012 (unit μas).

Year	MJD	X_C	X_S	S_X
1984.0	45700.0	4.6	−36.6	19.7
1985.0	46066.0	−141.8	−105.4	11.1
1986.0	46431.0	−246.6	−170.2	9.5
1987.0	46796.0	−281.9	−159.2	8.7
1988.0	47161.0	−255.1	−43.6	8.1
1989.0	47527.0	−210.5	−88.6	7.3
1990.0	47892.0	−187.8	−57.4	6.4
1991.0	48257.0	−163.0	26.3	5.5
1992.0	48622.0	−145.5	44.6	4.8
1993.0	48988.0	−145.1	51.5	5.9
1994.0	49353.0	−109.9	16.9	9.4
1995.0	49718.0	−87.3	5.4	8.3
1996.0	50083.0	−90.6	1.5	7.7
1997.0	50449.0	−94.7	35.3	4.4
1998.0	50814.0	−67.5	27.6	3.4
1999.0	51179.0	−44.1	−14.3	3.4
2000.0	51544.0	5.2	−74.9	3.3
2001.0	51910.0	70.4	−129.7	2.9
2002.0	52275.0	86.5	−127.8	2.8
2003.0	52640.0	110.4	−42.7	2.6
2004.0	53005.0	114.8	−0.1	2.5
2005.0	53371.0	133.0	−4.8	2.7
2006.0	53736.0	157.4	28.6	2.2
2007.0	54101.0	160.4	58.9	1.9
2008.0	54466.0	156.8	101.2	1.7
2009.0	54832.0	143.0	143.0	1.9
2010.0	55197.0	33.7	184.5	1.9
2011.0	55562.0	0.8	253.7	1.1
2012.0	55927.0	17.5	269.8	1.3

by the IERS to describe the most currently observed FCN motion of the CIP. Successive versions are identified by a date included in their name, the current realization (`fcnnut120701`) has been established on July 1st, 2012. Since 2011, the realization is fitted to the IERS 08 C04 data set which replaced the IERS 05 C04 data set on February 1st, 2011. Some significant changes exist with respect to adjustments made until 2010 (Lambert, 2011).

The model describes the quantities to be added to the X, Y coordinates of the CIP in the GCRS to account for the FCN effect. It provides this information in the form of a time-varying sinusoidal representation that assumes a constant period of $P = -430.21$ days. The equations for the model are

$$
\begin{aligned}
X_{FCN} &= X_S \sin(\sigma t) + X_C \cos(\sigma t), \\
Y_{FCN} &= Y_S \sin(\sigma t) + Y_C \cos(\sigma t),
\end{aligned}
\tag{5.28}
$$

where σ is the angular frequency of the FCN ($= 2\pi/P$ rad/d) and t is given in days since J2000.0.

Table 5.2c provides the coefficients for the model `fcnnut120701`. Older versions and updates can be found at $<^5>$. X_C and X_S are the amplitudes of the cosine and sine terms, respectively, and are piecewise defined in time. S_X is the formal error of the amplitude estimates. All amplitudes are in units of microarcseconds. Note that the values for Y_S and Y_C can be determined from the relationships $Y_S = -X_C$ and $Y_C = X_S$.

Over the period for which the model has been determined, the model should provide accuracies

better than 0.05 mas rms while for the period of extrapolation the model should provide accuracies better than 0.1 mas rms until the next annual update is provided.

Note that the unmodeled FCN motion of the CIP is included in the published IERS celestial pole offsets dX and dY. These offsets should not be applied when the FCN model is used.

5.5.6 Position of the Celestial Intermediate Origin in the GCRS

The numerical development of the quantity s (*i.e.* the CIO locator) appearing in Eq. (5.10), compatible with the IAU 2006/2000A precession-nutation model as well as the corresponding celestial offset at J2000.0, has been derived in a way similar to that based on the IERS Conventions 2003 (Capitaine *et al.*, 2003b). It results from expression (5.8) for s using the developments of X and Y as functions of time given by (5.16) (Capitaine *et al.*, 2003a). The numerical development is provided for the quantity $s + XY/2$, which requires fewer terms to reach the same accuracy than a direct development for s.

Table 5.2d: Development of $s(t)$ compatible with the IAU 2006/2000A precession-nutation model: all terms exceeding $0.5\,\mu$as during the interval 1975–2025 (unit μas). The expressions for the fundamental arguments appearing in column 1 are given by Eq. (5.43).

$$s(t) \;=\; -XY/2 + 94 + 3808.65t - 122.68t^2 - 72574.11t^3 + \sum_k C_k \sin\alpha_k$$
$$+1.73t\sin\Omega + 3.57t\cos 2\Omega + 743.52t^2\sin\Omega + 56.91t^2\sin(2F - 2D + 2\Omega)$$
$$+9.84t^2\sin(2F + 2\Omega) - 8.85t^2\sin 2\Omega$$

Argument α_k	Amplitude C_k
Ω	-2640.73
2Ω	-63.53
$2F - 2D + 3\Omega$	-11.75
$2F - 2D + \Omega$	-11.21
$2F - 2D + 2\Omega$	$+4.57$
$2F + 3\Omega$	-2.02
$2F + \Omega$	-1.98
3Ω	$+1.72$
$l' + \Omega$	$+1.41$
$l' - \Omega$	$+1.26$
$l + \Omega$	$+0.63$
$l - \Omega$	$+0.63$

The constant term for s, which was previously chosen so that $s(\mathrm{J2000.0}) = 0$, was subsequently fitted (Capitaine *et al.*, 2003b) so as to ensure continuity of UT1 at the date of change (1 January 2003) consistent with the Earth Rotation Angle (ERA) relationship and the current VLBI procedure for estimating UT1 (see (5.31)).

The complete series for $s + XY/2$ with all terms larger than $0.1\,\mu$as is available electronically on the IERS Conventions Center website $<^2>$ in file `tab5.2d.txt` and the terms larger than $0.5\,\mu$as over 25 years in the development of s are provided in Table 5.2d with μas accuracy. (There is no term where the change from IAU 2000 precession to IAU 2006 precession causes a change in s larger than $1\,\mu$as after one century.)

5.5.7 ERA based expressions for Greenwich Sidereal Time

Greenwich Sidereal Time (GST), which refers to the equinox, is related to the Earth Rotation Angle (ERA), that refers to the Celestial Intermediate Origin (CIO), by the following relationship (Aoki and Kinoshita, 1983; Capitaine and Gontier, 1993) at the microarcsecond level:

$$\mathrm{GST} = dT_0 + \mathrm{ERA} + \int_{t_0}^{t} (\dot{\widehat{\psi_A + \Delta\psi_1}})\cos(\omega_A + \Delta\epsilon_1)dt - \chi_A + \Delta\psi\cos\epsilon_A + \Delta\psi_1\cos\omega_A, \quad (5.29)$$

$\Delta\psi_1$, $\Delta\epsilon_1$, given by (5.24), being the nutation angles in longitude and obliquity referred to the ecliptic of epoch and χ_A, whose development is given in (5.40), the precession of the ecliptic along the equator (*i.e.* the right ascension component of the precession of the ecliptic).

This can be written as:

$$\text{GST} = \text{ERA(UT1)} - \text{EO}, \qquad (5.30)$$

where EO is the "equation of the origins", defined by:

$$\text{EO} = -dT_0 - \int_{t_0}^{t} (\dot{\widehat{\psi_A + \Delta\psi_1}}) \cos(\omega_A + \Delta\epsilon_1) dt + \chi_A - \Delta\psi \cos\epsilon_A + \Delta\psi_1 \cos\omega_A, \quad (5.31)$$

which is the CIO based right ascension of the equinox along the moving equator. The EO accounts for the accumulated precession and nutation in right ascension from J2000.0 to the date t; the constant term dT_0 was chosen to ensure continuity in UT1 at the date of change. A numerical expression for EO consistent with the IAU 2006/2000A precession-nutation model was provided by Capitaine *et al.* (2003c). The expression was obtained using computations similar to those performed for s and following a procedure, described below, that ensured consistency at the microarcsecond level among the transformations, as well as continuity in UT1 at the date of change (Capitaine *et al.*, 2003b).

The full series providing the expression for Greenwich Sidereal Time based on the IAU 2006/2000A precession-nutation model are available on the IERS Conventions Center website <2> in file `tab5.2e.txt`; the terms larger than $0.5\,\mu$as over 25 years in the development of the EO are provided in Table 5.2e. to 0.01 microarcsecond accuracy (*i.e.* with two digits).

Table 5.2e: Development of EO compatible with IAU 2006/2000A precession-nutation model: all terms exceeding $0.5\,\mu$as during the interval 1975–2025 (unit μas).

$\text{EO} \;=\; -0.014506'' - 4612.156534''t - 1.3915817''t^2$	
$+\, 0.00000044''t^3 - \Delta\psi\cos\epsilon_A \;-\; \sum_k C'_k \sin\alpha_k$	
Argument α_k	**Amplitude C'_k**
Ω	$+2640.96$
2Ω	$+63.52$
$2F - 2D + 3\Omega$	$+11.75$
$2F - 2D + \Omega$	$+11.21$
$2F - 2D + 2\Omega$	-4.55
$2F + 3\Omega$	$+2.02$
$2F + \Omega$	$+1.98$
3Ω	-1.72
$l' + \Omega$	-1.41
$l' - \Omega$	-1.26
$l + \Omega$	-0.63
$l - \Omega$	-0.63

The C'_k coefficients are similar to the C_k coefficients appearing in Table 5.2d providing the development for s, and are equal to these coefficients up to $1\,\mu$as. The last term in the EO expression, *i.e.* $-\sum_k C'_k \sin\alpha_k$, comprises the complementary terms to be subtracted from the classical "equation of the equinoxes," $\Delta\psi\cos\epsilon_A$, to provide the relationship between GST and ERA with microarcsecond accuracy. These were introduced in the IERS Conventions (2003), replacing the two complementary terms provided in the IERS Conventions (1996). A secular term similar to that appearing in the quantity s is included in expression (5.31). This expression for GST used in the equinox based transformation ensures consistency with the CIO based transformation at the microarcsecond level after one century, using expressions (5.14) for ERA, (5.16) for the celestial coordinates of the CIP and Table 5.2d for s.

As an alternative to using the developments set out in Table 5.2e, the EO can be calculated from the quantity s and the classical (*i.e.* equinox based) nutation $\times$ precession $\times$ bias matrix: see Wallace and Capitaine (2006), Eq. (5.16).

The numerical values chosen for the constant term dT_0 in GST to achieve continuity in UT1 at the transition date (1 January 2003), and for the corresponding constant term in s are $dT_0 = +14506\,\mu$as and $s_0 = +94\,\mu$as. The polynomial part of GST (*i.e.* of ERA(UT1) $-$ EO(t)) provides the IAU 2006 expression for Greenwich Mean Sidereal Time, GMST (Capitaine *et al.*, 2003c):

$$
\begin{aligned}
\text{GMST} \;=\; & \text{ERA(UT1)} + 0.014506'' + 4612.156534''t + 1.3915817''t^2 - 0.00000044''t^3 \\
& - 0.000029956''t^4 - 0.0000000368''t^5 \,.
\end{aligned}
\tag{5.32}
$$

This expression for GMST clearly distinguishes between ERA, which is expressed as a function of UT1, and the EO (*i.e.* mainly the accumulated precession-nutation in right ascension), which is expressed as a function of TDB (or, in practice, TT), in contrast to the GMST_{1982}(UT1) expression (Aoki *et al.*, 1982), which used only UT1. The difference between these two processes gives rise to a term in GMST of (TT$-$UT1) multiplied by the speed of precession in right ascension. Using TT$-$TAI = 32.184 s, this was: $[47+1.5(\text{TAI}-\text{UT1})]\,\mu$as, where TAI$-$UT1 is in seconds. On 1 January 2003, this difference was about $94\,\mu$as (see Gontier in Capitaine *et al.*, 2002), using the value of 32.3 s for TAI$-$UT1. This is included in the values for dT_0 and s_0.

5.6 Description of the IAU 2000/2006 precession-nutation model

The following sections describe the main features of the IAU 2006/2000 precession-nutation. Comparisons of the IAU 2000/2006 precession-nutation with other models and VLBI observations can be found in Capitaine *et al.* (2009).

5.6.1 The IAU 2000A and IAU 2000B nutation model

The IAU 2000A model, developed by Mathews *et al.,* (2002) and denoted MHB2000, is based on the REN2000 rigid Earth nutation series of Souchay *et al.* (1999) for the axis of figure (available at: `ftp://syrte.obspm.fr/pub/REN2000/`). The REN2000 solution is provided as a series of luni-solar and planetary nutations in longitude and obliquity, referred to the ecliptic of date, expressed as "in-phase" and "out-of-phase" components with their time variations. The sub-diurnal terms arising for the imperfect axial symmetry of the Earth are not part of this solution, so that the axis of reference of the nutation model be compliant with the definition of the CIP.

The rigid Earth nutation was transformed to the non-rigid Earth nutation by applying the MHB2000 "transfer function" to the full REN2000 series of the corresponding prograde and retrograde nutations and then converting back into elliptical nutations. This "transfer function" is based on the solution of the linearized dynamical equation of the wobble-nutation problem and makes use of estimated values of seven of the parameters appearing in the theory (called "Basic Earth Parameters" (BEP)), obtained from a least-squares fit of the theory to an up-to-date precession-nutation VLBI data set (Herring *et al.*, 2002). The estimation of the dynamical ellipticity of the Earth resulting in a value differing slightly from that of the reference rigid Earth series (*i.e.* by the multiplying factor 1.000012249) was a part of the estimation of the parameters needed for the non-rigid Earth series. The MHB2000 model improves the IAU 1980 theory of nutation by taking into account the effect of mantle anelasticity, ocean tides, electromagnetic couplings produced between the fluid outer core and the mantle as well as between the solid inner core and fluid outer core (Buffett *et al.*, 2002) and the consideration of nonlinear terms which have hitherto been ignored in this type of formulation. The axis of reference is the axis of maximum moment of inertia of the Earth in steady rotation (*i.e.* ignoring time-dependent deformations).

The resulting nutation series includes 678 lunisolar terms and 687 planetary terms which are expressed as "in-phase" and "out-of-phase" components with their time variations (see expression(5.35)). That model is expected to have an accuracy of about $10\,\mu$as for most of its terms.

On the other hand, the FCN, being a free motion which cannot be predicted rigorously (see Section 5.5.5), is not considered a part of the IAU 2000A model, which limits the accuracy in the computed direction of the celestial pole in the GCRS to about 0.3 mas.

The IAU 2000A nutation series is associated with the following offset (originally provided as frame bias in $\mathrm{d}\psi_{bias}$ and $\mathrm{d}\epsilon_{bias}$) of the direction of the CIP at J2000.0 from the direction of the pole of the GCRS:

$$
\begin{array}{rcl}
\xi_0 & = & (-0.0166170 \pm 0.0000100)'', \\
\eta_0 & = & (-0.0068192 \pm 0.0000100)''.
\end{array}
\tag{5.33}
$$

The IAU 2000A nutation includes the geodesic nutation contributions to the annual, semiannual and 18.6-year terms to be consistent with including the geodesic precession ($p_g = 1.92''$/century) in the precession model and so that the BCRS and GCRS are without any time-dependent rotation. The theoretical geodesic nutation contribution (Fukushima, 1991) used in the MHB2000 model (Mathews $et\ al.$, 2002) is, in μas, for the nutations in longitude $\Delta\psi_g$ and obliquity $\Delta\epsilon_g$

$$
\begin{array}{rcl}
\Delta\psi_g & = & -153\ \sin l' - 2\ \sin 2l' + 3\ \sin\Omega, \\
\Delta\epsilon_g & = & 1\ \cos\Omega,
\end{array}
\tag{5.34}
$$

where l' is the mean anomaly of the Sun and Ω the longitude of the ascending node of the Moon.

The IAU 2000 nutation model is given by a series for nutation in longitude $\Delta\psi$ and obliquity $\Delta\epsilon$, referred to the ecliptic of date, with t measured in Julian centuries from epoch J2000.0:

$$
\begin{array}{rcl}
\Delta\psi & = & \sum_{i=1}^{N}(A_i + A_i' t)\sin(\textsc{argument}) + (A_i'' + A_i''' t)\cos(\textsc{argument}), \\
\Delta\epsilon & = & \sum_{i=1}^{N}(B_i + B_i' t)\cos(\textsc{argument}) + (B_i'' + B_i''' t)\sin(\textsc{argument}).
\end{array}
\tag{5.35}
$$

The IAU 2000 resolution on precession-nutation (B1.6) adopted the MHB2000 model which, as provided by the authors at that time (in the form of Fortran code), did not contain the A''' and B''' terms in Eq. (5.35). The SOFA implementation, NUT00A, adopted that version of the model. The differences between the SOFA and IERS implementations of the model are insignificant ($<$ $1\,\mu$as) for practical purposes. More details about the coefficients and arguments of these series will be given in Section 5.7.

The IERS Conventions (2003) implementation of the IAU 2000A nutation series are available electronically on the IERS Conventions Center website at $<^6>$ in the files tab5.3a.txt and tab5.3b.txt for the lunisolar and planetary components, respectively. The "total nutation" includes all components. Note that there was a sign error in Table 5.3b in the Conventions (2003), resulting in errors of several hundred μas for the planetary nutations (Wallace, 2011).

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The series corresponding to nutation "IAU 2000A$_{R06}$" (see Section 5.6.3 for the definition of the subscript "R06") are available electronically at $<^4>$. They are provided by the files tab5.3a.txt (Table 5.3a) and tab5.3b.txt (Table 5.3b), for nutation in longitude $\Delta\psi$ and obliquity $\Delta\epsilon$, respectively. An extract from Tables 5.3a and 5.3b for the largest nutation components is given below. (Note that the headings of those files as well as the caption below have changed with respect to the IERS Conventions (2003) available at $<^6>$.)

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As recommended by IAU 2000 Resolution B1.6, an abridged model, designated IAU 2000B, is available for those who need a model only at the 1 mas level. Such a model has been developed by McCarthy and Luzum (2003). It includes fewer than 80 lunisolar terms plus a bias to account for the effect of the planetary terms in the time period under consideration. It provides the celestial pole motion with an accuracy that does not result in a difference greater than 1 mas with respect to that of the IAU 2000A model during the period 1995–2050. The model is implemented in the Fortran subroutine IAU2000B.f, available electronically on the IERS Conventions Center website at $<^2>$. SOFA provides IAU00B as well.

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[6] ftp://tai.bipm.org/iers/conv2003/chapter5/

Table 5.3a: Extract from Table 5.3a (nutation in longitude) (available at <2>) providing the largest components for the "in-phase" and "out-of-phase" terms of the "IAU 2000A$_{\mathrm{R06}}$" nutation. Units are μas and μas/century for the coefficients and their time variations, respectively. The expressions for the Delaunay arguments appearing in columns 1 to 5 are given by Eq. (5.43); those for the additional fundamental arguments of the planetary nutations appearing in columns 6 to 14 (bottom part) are given by Eq. (5.44). (Because the largest terms are all luni-solar, columns 9-17 contain only zeros in the extract shown.)

i	A_i	A_i''	l	l'	F	D	Ω	L_{Me}	L_{Ve}	L_E	L_{Ma}	L_J	L_{Sa}	L_U	L_{Ne}	p_A
1	−17206424.18	3338.60	0	0	0	0	1	0	0	0	0	0	0	0	0	0
2	−1317091.22	−1369.60	0	0	2	−2	2	0	0	0	0	0	0	0	0	0
3	−227641.81	279.60	0	0	2	0	2	0	0	0	0	0	0	0	0	0
4	207455.50	−69.80	0	0	0	0	2	0	0	0	0	0	0	0	0	0
5	147587.77	1181.70	0	1	0	0	0	0	0	0	0	0	0	0	0	0
.....																
i	A_i'	A_i'''	l	l'	F	D	Ω	L_{Me}	L_{Ve}	L_E	L_{Ma}	L_J	L_{Sa}	L_U	L_{Ne}	p_A
1321	− 17418.82	2.89	0	0	0	0	1	0	0	0	0	0	0	0	0	0
1322	−363.71	-1.50	0	1	0	0	0	0	0	0	0	0	0	0	0	0
1323	−163.84	1.20	0	0	2	−2	2	0	0	0	0	0	0	0	0	0
.....																

Table 5.3b: Extract from Table 5.3b (nutation in obliquity) (available at <2>) providing the largest components for the "in-phase" and "out-of-phase" terms of the "IAU 2000A$_{\mathrm{R06}}$" nutation. Units are μas and μas/century for the coefficients and their time variations, respectively. The expressions for the Delaunay arguments appearing in columns 1 to 5 are given by Eq. (5.43); those for the additional fundamental arguments of the planetary nutations appearing in columns 6 to 14 (bottom part) are given by Eq. (5.44). (Because the largest terms are all luni-solar, columns 9-17 contain only zeros in the extract shown.)

i	B_i''	B_i	l	l'	F	D	Ω	L_{Me}	L_{Ve}	L_E	L_{Ma}	L_J	L_{Sa}	L_U	L_{Ne}	p_A
1	1537.70	9205233.10	0	0	0	0	1	0	0	0	0	0	0	0	0	0
2	−458.70	573033.60	0	0	2	−2	2	0	0	0	0	0	0	0	0	0
3	137.40	97846.10	0	0	2	0	2	0	0	0	0	0	0	0	0	0
4	−29.10	−89749.20	0	0	0	0	2	0	0	0	0	0	0	0	0	0
5	−17.40	22438.60	0	1	2	−2	2	0	0	0	0	0	0	0	0	0
.....																
i	B_i'''	B_i'	l	l'	F	D	Ω	L_{Me}	L_{Ve}	L_E	L_{Ma}	L_J	L_{Sa}	L_U	L_{Ne}	p_A
1038	0.20	883.03	0	0	0	0	1	0	0	0	0	0	0	0	0	0
1039	−0.30	−303.09	0	0	2	−2	2	0	0	0	0	0	0	0	0	0
1040	0.00	−67.76	0	1	2	−2	2	0	0	0	0	0	0	0	0	0
.....																

5.6.2 Description of the IAU 2006 precession

The IAU 2006 precession (Capitaine *et al.*, 2003c) provides improved polynomial expressions up to the 5th degree in time t, both for the precession of the ecliptic and the precession of the equator, the latter being consistent with dynamical theory while matching the IAU 2000A precession rate for continuity reasons.

While the precession part of the IAU 2000A model consists only of corrections ($\delta\psi_A = (-0.29965 \pm 0.00040)''$/century, $\delta\omega_A = (-0.02524 \pm 0.00010)''$/century) to the precession rates of the IAU 1976 precession, the IAU 2006 precession of the equator was derived from the dynamical equations expressing the motion of the mean pole about the ecliptic pole, with $\epsilon_0 = 84381.406''$ for the mean obliquity at J2000.0 of the ecliptic (while the IAU 2000 value was $84381.448''$). The IAU 2006 value for the Earth's dynamical flattening is quite consistent with the IAU 2000 value; the IAU 2006 precession includes the Earth's J_2 rate effect (*i.e.* $\dot{J}_2 = -3 \times 10^{-9}$/century), mostly due to the post-glacial rebound, which was not taken into account in IAU 2000. The other contributions to

the IAU 2006 precession rates are from Williams (1994) and MHB2000, and the geodesic precession is from Brumberg (1991). These also include corrections for the perturbing effects in the observed quantities.

5.6.3 IAU 2006 adjustments to the IAU 2000A nutation

The difference between IAU 2006 and IAU 2000 lies essentially in the precession part, though very small changes are needed in a few of the IAU 2000A nutation amplitudes in order to ensure compatibility with the IAU 2006 values for ϵ_0 and the J_2 rate.

- Introducing the IAU 2006 J_2 rate value gives rise to additional Poisson terms in nutation, the coefficients of which are proportional to $\dot{J}_2/J_2$ (*i.e.* -2.7774×10^{-6}/century). The largest changes (*cf.* (5.19) for the corresponding changes in the X, Y series) in μas are (Capitaine and Wallace, 2006):

$$
\begin{aligned}
d\psi_{J2d} &= +47.8\,t\,\sin\Omega + 3.7\,t\,\sin 2(F - D + \Omega) + 0.6\,t\,\sin 2(F + \Omega) - 0.6\,t\,\sin 2\Omega, \\
d\varepsilon_{J2d} &= -25.6\,t\,\cos\Omega - 1.6\,t\,\cos 2(F - D + \Omega).
\end{aligned}
\tag{5.36}
$$

- The effect of the adjustment to the IAU 2006 value for ϵ_0 results from the fact that the IAU 2006 obliquity is different from the IAU 1980 obliquity that was used when estimating the IAU 2000A nutation amplitudes. To compensate for this change, it is necessary to multiply the amplitudes of the nutation in longitude by $\sin\epsilon_{\mathrm{IAU2000}}/\sin\epsilon_{\mathrm{IAU2006}} = 1.000000470$. (No such adjustment is needed in the case of X, Y.)

  The largest terms in the correction applied to the IAU 2000A nutation in longitude for this effect are, in μas:
  $$
  d_\epsilon\psi = -8.1\,\sin\Omega - 0.6\,\sin 2(F - D + \Omega).
  \tag{5.37}
  $$

Whenever these small adjustments are included, the notation "IAU 2000A$_{\mathrm{R06}}$" can be used to indicate that the nutation has been revised for use with the IAU 2006 precession. The adjustments are taken into account in the SOFA implementation of the IAU 2006/2000A precession-nutation (Section 5.9). The difference between the IAU 2000A$_{\mathrm{R06}}$ and IAU 2000 expressions for the nutation in longitude and obliquity is available electronically on the IERS Conventions Center website (Table 5.2f) at $<^2>$ in file `tab5.2f.txt`.

5.6.4 Precession developments compatible with the IAU 2000/2006 model

The IAU 2006 precession polynomial developments (Capitaine *et al.*, 2003c) provide separately the developments for the basic quantities for the ecliptic and the equator that are direct solutions of the dynamical equations, and derived quantities, such as those for the GCRS coordinates of the CIP, X, Y, which were given by (5.16), or for sidereal time (see Section 5.5.7). The latter can be obtained from the expression for the Earth Rotation Angle, which is independent of the precession-nutation model, and the expression for the equation of the origins (*i.e.* the distance between the CIO and the equinox along the CIP equator), which is directly model-dependent.

The basic expressions for the precession of the ecliptic and the equator are provided by (5.38) and (5.39), respectively, with $\epsilon_0 = 84381.406''$:

$$
\begin{aligned}
P_A &= +4.199094''\,t + 0.1939873''\,t^2 - 0.00022466''\,t^3 - 0.000000912''\,t^4 + 0.0000000120''\,t^5 \\
Q_A &= -46.811015''\,t + 0.0510283''\,t^2 + 0.00052413''\,t^3 - 0.000000646''\,t^4 - 0.0000000172''\,t^5
\end{aligned}
\tag{5.38}
$$

$$
\begin{aligned}
\psi_A &= 5038.481507''\,t - 1.0790069''\,t^2 - 0.00114045\,t^3 + 0.000132851''\,t^4 - 0.0000000951''\,t^5 \\
\omega_A &= \epsilon_0 - 0.025754''\,t + 0.0512623''\,t^2 - 0.00772503''\,t^3 - 0.000000467''\,t^4 + 0.0000003337''\,t^5
\end{aligned}
\tag{5.39}
$$

The precession quantities $P_A = \sin \pi_A \sin \Pi_A$ and $Q_A = \sin \pi_A \cos \Pi_A$ are the secular parts of the developments of the quantities $P = \sin \pi \sin \Pi$ and $Q = \sin \pi \cos \Pi$, π and Π being the osculating elements (*i.e.* the inclination and longitude of the ascending node, respectively) of the Earth-Moon barycenter orbit referred to the fixed ecliptic for J2000.0. The precession quantities ψ_A and ω_A, defined in Section 5.4.5, are solutions of the dynamical equations expressing the motion of the mean pole about the ecliptic pole.

Derived expressions for other precession quantities are given below:

$$
\begin{aligned}
\chi_A &= 10.556403'' \, t - 2.3814292'' \, t^2 - 0.00121197'' \, t^3 \\
&\quad + 0.000170663'' \, t^4 - 0.0000000560'' \, t^5, \\
\epsilon_A &= \epsilon_0 - 46.836769'' \, t - 0.0001831'' \, t^2 + 0.00200340'' \, t^3 \\
&\quad - 0.000000576'' \, t^4 - 0.0000000434'' \, t^5, \\
\bar{\gamma} &= -0.052928'' + 10.556378'' \, t + 0.4932044'' \, t^2 - 0.00031238'' \, t^3 \\
&\quad - 0.000002788'' \, t^4 + 0.0000000260'' \, t^5, \\
\bar{\phi} &= +84381.412819'' - 46.811016'' \, t + 0.0511268'' \, t^2 + 0.00053289'' \, t^3 \\
&\quad - 0.000000440'' \, t^4 - 0.0000000176'' \, t^5, \\
\bar{\psi} &= -0.041775'' + 5038.481484'' \, t + 1.5584175'' \, t^2 - 0.00018522'' \, t^3 \\
&\quad - 0.000026452'' \, t^4 - 0.0000000148'' \, t^5,
\end{aligned}
\tag{5.40}
$$

where χ_A is the precession of the ecliptic along the equator and ϵ_A is the mean obliquity of date; $\bar{\gamma}$, $\bar{\phi}$ and $\bar{\psi}$ are the angles (Williams 1994, Fukushima 2003) referred to the GCRS. $\bar{\gamma}$ is the GCRS right ascension of the intersection of the ecliptic of date with the GCRS equator, $\bar{\phi}$ is the obliquity of the ecliptic of date on the GCRS equator and $\bar{\psi}$ is the precession angle plus bias in longitude along the ecliptic of date. The last three series are from Table 1 of Hilton *et al.* (2006).

Due to their theoretical bases, the original development of the precession quantities as functions of time can be considered as being expressed in TDB. However, in practice, TT is used in the above expressions in place of TDB. The largest term in the difference TDB−TT being 1.7 ms $\times \sin l'$ (where l' is the mean anomaly of the Sun), the resulting error in the precession quantity ψ_A is periodic, with an annual period and an amplitude of $2.7'' \times 10^{-9}$, which is significantly below the required microarcsecond accuracy. This applies to Eq. (5.38)-(5.40), as well as to the polynomial part of Eq. (5.16) (*i.e.* the expression for the CIP's GCRS X, Y coordinates).

5.6.5 Summary of the different ways of implementing the IAU 2006/2000A precession-nutation

There are several ways to implement the precession-nutation model, and the precession developments to be used should be consistent with the procedure being used.

Using the CIO based paradigm, the complete procedure to transform between the ITRS and the GCRS compatible with the IAU 2006/2000A precession-nutation is based on the IAU 2000 expression (*i.e.* Eq. (5.15)) for the ERA and on expressions provided by (5.16) and Tables 5.2a–5.2b and Table 5.2d for the positions of the CIP and the CIO in the GCRS. These already contain the proper expressions for the new precession-nutation model and the frame biases.

Implementing the IAU 2006/2000A/B model using the equinox based transformation between the ITRS and the GCRS requires following one of the rigorous procedures mentioned in Section 5.4.5 that are supported by SOFA routines (Section 5.9). They are based upon the IAU 2000 bias, IAU 2006 precession (*cf.* Eq. (5.39)) and the (adjusted) IAU 2000A nutation. These transformations should be used in conjunction with the IAU 2006 expression for Greenwich Sidereal Time (*cf.* Eq. (5.31) and Table 5.2e).

5.7 The fundamental arguments of nutation theory

5.7.1 The multipliers of the fundamental arguments of nutation theory

Each of the lunisolar terms in the nutation series is characterized by a set of five integers N_j which determines the argument for the term as a linear combination of the five Fundamental Arguments

F_j, namely the Delaunay variables (l, l', F, D, Ω): ARGUMENT $= \sum_{j=1}^{5} N_j F_j$, where the values $(N_1, \cdots, N_5)$ of the multipliers characterize the term. The F_j are functions of time, and the angular frequency of the nutation described by the term is given by

$$\omega \equiv d(\text{ARGUMENT})/dt. \tag{5.41}$$

The frequency thus defined is positive for most terms, and negative for some. Planetary nutation terms differ from the luni-solar nutation terms only in the fact that ARGUMENT $= \sum_{j=1}^{14} N_j' F_j'$, F_6 to F_{13}, as noted in Tables 5.2a-5.2b and 5.3b, are the mean longitudes of the planets including the Earth (L_{Me}, L_{Ve}, L_E, L_{Ma}, L_J, L_{Sa}, L_U, l_{Ne}) and F_{14} is the general precession in longitude p_A.

Over time scales involved in nutation studies, the frequency ω is effectively time-independent, and one may write, for the kth term in the nutation series,

$$\text{ARGUMENT} = \omega_k t + \beta_k. \tag{5.42}$$

Tables of IAU 2000 nutation provide, for each (ARGUMENT), the coefficients of the "in phase" and "out-of-phase" components in longitude $\Delta\psi$ and obliquity $\Delta\epsilon$, plus their time variations in the case of the luni-solar nutations.

Different tables of nutations in longitude and obliquity do not necessarily assign the same set of multipliers N_j to a particular term in the nutation series. The differences in the assignments arise from the fact that the replacement $(N_{j=1,14}) \rightarrow -(N_{j=1,14})$ accompanied by reversal of the sign of the coefficient of sin(ARGUMENT) in the series for $\Delta\psi$ and $\Delta\epsilon$ leaves these series unchanged.

In the original expressions for the fundamental arguments F_1-F_{14} of luni-solar and planetary nutations as functions of time, t is measured in TDB. However, the changes in the nutation amplitudes resulting from the contributions, $\omega_k(\text{TDB}-\text{TT})$, of the difference TDB$-$TT (whose largest term is 1.7 ms $\times \sin l'$) to the nutation arguments $(\omega_k t + \beta_k)$ are responsible for a difference in the CIP location that is less than 0.01 μas, which is significantly below the required microarcsecond accuracy. Consequently, TT can be used in practice in place of TDB in the expressions for the fundamental nutation arguments, as it is the case for the precession expressions (*cf.* Section 5.6.4). This also applies to the non-polynomial part of Eq. (5.16) for the GCRS CIP coordinates.

5.7.2 Development of the arguments of lunisolar nutation

The expressions for the fundamental arguments of nutation are given by the following developments where t is measured in Julian centuries of TDB (Simon *et al.*, 1994: Tables 3.4 (b.3) and 3.5 (b)) based on IERS 1992 constants and Williams *et al.* (1991), for precession.

$$\begin{aligned}
F_1 \equiv l \ &= \ Mean \ Anomaly \ of \ the \ Moon \\
&= 134.96340251° + 1717915923.2178''t + 31.8792''t^2 \\
&\quad + 0.051635''t^3 - 0.00024470''t^4, \\[6pt]
F_2 \equiv l' \ &= \ Mean \ Anomaly \ of \ the \ Sun \\
&= 357.52910918° + 129596581.0481''t - 0.5532''t^2 \\
&\quad + 0.000136''t^3 - 0.00001149''t^4, \\[6pt]
F_3 \equiv F \ &= L - \Omega \\
&= 93.27209062° + 1739527262.8478''t - 12.7512''t^2 \\
&\quad - 0.001037''t^3 + 0.00000417''t^4, \\[6pt]
F_4 \equiv D \ &= \ Mean \ Elongation \ of \ the \ Moon \ from \ the \ Sun \\
&= 297.85019547° + 1602961601.2090''t - 6.3706''t^2 \\
&\quad + 0.006593''t^3 - 0.00003169''t^4, \\[6pt]
F_5 \equiv \Omega \ &= \ Mean \ Longitude \ of \ the \ Ascending \ Node \ of \ the \ Moon \\
&= 125.04455501° - 6962890.5431''t + 7.4722''t^2 \\
&\quad + 0.007702''t^3 - 0.00005939''t^4
\end{aligned} \tag{5.43}$$

where L is the Mean Longitude of the Moon.

Note that the SOFA implementation is the IAU 2000A nutation, as defined by the published MHB2000 code (T. Herring 2002). As part of this strict compliance, SOFA uses the original MHB2000 expressions for the Delaunay variables l' and D, that differ from Eq. (5.43) in that the fixed term is rounded to five digits (*i.e.* $1287104.79305''$ instead of $1287104.793048''$ for the Eq. (5.43) value in the l expression converted into arcseconds and $1072260.70369''$ instead of $1072260.703692''$ for the Eq. (5.43) value in the l expression converted into arcseconds), respectively. The CIP location is insensitive to this difference of 2 μas in the nutation arguments at a level better than 10^{-9} arcsec accuracy.

It should also be noted that the SOFA implementation of the IAU 2000A nutation follows the MHB2000 Fortran code in not including time variations of the out of phase components, *i.e.* the A_i''' and B_i''' columns of Table 5.3a (see Section 5.6.1). The difference in the CIP location is just over 2 μas after one century.

5.7.3 Development of the arguments for the planetary nutation

The mean longitudes of the planets used in the arguments for the planetary nutations are essentially those provided by Souchay *et al.* (1999), based on theories and constants of VSOP82 (Bretagnon, 1982) and ELP 2000 (Chapront-Touzé and Chapront, 1983) and developments of Simon *et al.* (1994: Tables 5.8.1-5.8.8). Their developments are given in Eq. (5.44) in radians with t in Julian centuries.

The general precession, F_{14}, is from Kinoshita and Souchay (1990).

$$
\begin{aligned}
F_6 &\equiv L_{Me} &= 4.402608842 + 2608.7903141574 \times t, \\
F_7 &\equiv L_{Ve} &= 3.176146697 + 1021.3285546211 \times t, \\
F_8 &\equiv L_E &= 1.753470314 + 628.3075849991 \times t, \\
F_9 &\equiv L_{Ma} &= 6.203480913 + 334.0612426700 \times t, \\
F_{10} &\equiv L_J &= 0.599546497 + 52.9690962641 \times t, \\
F_{11} &\equiv L_{Sa} &= 0.874016757 + 21.3299104960 \times t, \\
F_{12} &\equiv L_U &= 5.481293872 + 7.4781598567 \times t, \\
F_{13} &\equiv L_{Ne} &= 5.311886287 + 3.8133035638 \times t, \\
F_{14} &\equiv p_A &= 0.02438175 \times t + 0.00000538691 \times t^2.
\end{aligned}
\tag{5.44}
$$

Another part of the strict compliance of SOFA with the MHB2000 code is that simplified expressions are used for the Delaunay variables F_1-F_5 in the planetary nutation case. The maximum difference this makes to the CIP location is 0.013 μas after one century. The SOFA implementation also uses the original MHB2000 expression for the longitude of Neptune (slightly different from that in Eq. (5.44), *i.e.* $L_{Ne} = 5.321159000 + 3.812777400 \times t$ instead of $L_{Ne} = 5.311886287 + 3.8133035638 \times t$). The maximum difference in the CIP is less than 0.01 μas after one century.

5.8 Prograde and retrograde nutation amplitudes

The quantities $\Delta\psi(t)\sin\epsilon_0$ and $\Delta\epsilon(t)$ may be viewed as the components of a moving two-dimensional vector in the mean equatorial system, with the positive X and Y axes pointing along the directions of increasing $\Delta\psi$ and $\Delta\epsilon$, respectively. The purely periodic parts of $\Delta\psi(t)\sin\epsilon_0$ and $\Delta\epsilon(t)$ for a term of frequency ω_k are made up of in-phase and out-of-phase parts

$$
\begin{aligned}
(\Delta\psi^{ip}(t)\sin\epsilon_0, \ \Delta\epsilon^{ip}(t)) &= (\Delta\psi_k^{ip}\sin\epsilon_0\sin(\omega_k t + \beta_k), \ \Delta\epsilon_k^{ip}\cos(\omega_k t + \beta_k)), \\
(\Delta\psi^{op}(t)\sin\epsilon_0, \ \Delta\epsilon^{op}(t)) &= (\Delta\psi_k^{op}\sin\epsilon_0\cos(\omega_k t + \beta_k), \ \Delta\epsilon_k^{op}\sin(\omega_k t + \beta_k)),
\end{aligned}
\tag{5.45}
$$

respectively. Each of these vectors may be decomposed into two uniformly rotating vectors, one constituting a prograde circular nutation (rotating in the same sense as from the positive X axis

towards the positive Y axis) and the other a retrograde one rotating in the opposite sense. The decomposition is facilitated by factoring out the sign q_k of ω_k from the argument, q_k being such that

$$q_k \omega_k \equiv |\omega_k| \tag{5.46}$$

and writing

$$\omega_k t + \beta_k = q_k(|\omega_k| t + q_k \beta_k) \equiv q_k \chi_k, \tag{5.47}$$

with χ_k increasing linearly with time. The pair of vectors above then becomes

$$(\Delta\psi^{ip}(t) \sin\epsilon_0, \ \Delta\epsilon^{ip}(t)) = (q_k \Delta\psi_k^{ip} \sin\epsilon_0 \sin\chi_k, \ \Delta\epsilon_k^{ip} \cos\chi_k),$$
$$(\Delta\psi^{op}(t) \sin\epsilon_0, \ \Delta\epsilon^{op}(t)) = (\Delta\psi_k^{op} \sin\epsilon_0 \cos\chi_k, \ q_k \Delta\epsilon_k^{op} \sin\chi_k). \tag{5.48}$$

Because χ_k increases linearly with time, the mutually orthogonal unit vectors $(\sin\chi_k, -\cos\chi_k)$ and $(\cos\chi_k, \sin\chi_k)$ rotate in a prograde sense and the vectors obtained from these by the replacement $\chi_k \to -\chi_k$, namely $(-\sin\chi_k, -\cos\chi_k)$ and $(\cos\chi_k, -\sin\chi_k)$ are in retrograde rotation. On resolving the in-phase and out-of-phase vectors in terms of these, one obtains

$$(\Delta\psi^{ip}(t) \sin\epsilon_0, \ \Delta\epsilon^{ip}(t)) = A_k^{pro\ ip}(\sin\chi_k, \ -\cos\chi_k) + A_k^{ret\ ip}(-\sin\chi_k, \ -\cos\chi_k),$$
$$(\Delta\psi^{op}(t) \sin\epsilon_0, \ \Delta\epsilon^{op}(t)) = A_k^{pro\ op}(\cos\chi_k, \ \sin\chi_k) + A_k^{ret\ op}(\cos\chi_k, \ -\sin\chi_k), \tag{5.49}$$

where

$$\begin{aligned}
A_k^{pro\ ip} &= \tfrac{1}{2}\left(q_k \Delta\psi_k^{ip} \sin\epsilon_0 - \Delta\epsilon_k^{ip}\right), \\
A_k^{ret\ ip} &= -\tfrac{1}{2}\left(q_k \Delta\psi_k^{ip} \sin\epsilon_0 + \Delta\epsilon_k^{ip}\right), \\
A_k^{pro\ op} &= \tfrac{1}{2}\left(\Delta\psi_k^{op} \sin\epsilon_0 + q_k \Delta\epsilon_k^{op}\right), \\
A_k^{ret\ op} &= \tfrac{1}{2}\left(\Delta\psi_k^{op} \sin\epsilon_0 - q_k \Delta\epsilon_k^{op}\right).
\end{aligned} \tag{5.50}$$

The expressions providing the corresponding nutation in longitude and in obliquity from circular terms are

$$\begin{aligned}
\Delta\psi_k^{ip} &= \frac{q_k}{\sin\epsilon_0}\left(A_k^{pro\ ip} - A_k^{ret\ ip}\right), \\
\Delta\psi_k^{op} &= \frac{1}{\sin\epsilon_0}\left(A_k^{pro\ op} + A_k^{ret\ op}\right), \\
\Delta\epsilon_k^{ip} &= -\left(A_k^{pro\ ip} + A_k^{ret\ ip}\right), \\
\Delta\epsilon_k^{op} &= q_k\left(A_k^{pro\ op} - A_k^{ret\ op}\right).
\end{aligned} \tag{5.51}$$

The contribution of the k-term of the nutation to the position of the Celestial Intermediate Pole (CIP) in the mean equatorial system is thus given by the complex coordinate

$$\Delta\psi(t) \sin\epsilon_0 + i\Delta\epsilon(t) = -i\left(A_k^{pro} e^{i\chi_k} + A_k^{ret} e^{-i\chi_k}\right), \tag{5.52}$$

where A_k^{pro} and A_k^{ret} are the amplitudes of the prograde and retrograde components, respectively, and are given by

$$A_k^{pro} = A_k^{pro\ ip} + iA_k^{pro\ op}, \qquad A_k^{ret} = A_k^{ret\ ip} + iA_k^{ret\ op}. \tag{5.53}$$

The decomposition into prograde and retrograde components is important for studying the role of resonance in nutation because any resonance (especially in the case of the nonrigid Earth) affects A_k^{pro} and A_k^{ret} unequally.

In the literature (Wahr, 1981) one finds an alternative notation, frequently followed in analytic formulations of nutation theory, that is:

$$\Delta\epsilon(t) + i\Delta\psi(t) \sin\epsilon_0 = -i\left(A_k^{pro\ -} e^{-i\chi_k} + A_k^{ret\ -} e^{i\chi_k}\right), \tag{5.54}$$

with

$$A_k^{pro\ -} = A_k^{pro\ ip} - iA_k^{pro\ op}, \qquad A_k^{ret\ -} = A_k^{ret\ ip} - iA_k^{ret\ op}. \tag{5.55}$$

Further details can be found in Defraigne *et al.* (1995) and Bizouard *et al.* (1998).

5.9 Algorithms for transformations between ITRS and GCRS

Software routines to implement the IAU 2006/2000A transformations are provided by the IAU *Standards Of Fundamental Astronomy* service.[7] The routines vary in complexity from simple modules to complete transformations, allowing the application developer to trade off simplicity of use against computational efficiency and flexibility. Implementations in Fortran77 and C are available.

The SOFA software supports two equivalent ways of implementing the IAU resolutions in the transformation from ITRS to GCRS provided by expression (5.1), namely (a) the transformation based on the Celestial Intermediate Origin and the Earth Rotation Angle (*i.e.* the CIO based procedure described in Sections 5.4.1, 5.4.2 and 5.4.4, using parameters described in Section 5.5) and (b) the classical transformation based on the equinox and Greenwich Sidereal Time (*i.e.* the equinox based procedure described in Sections 5.4.1, 5.4.3 and 5.4.5, with the use of classical precession and nutation angles). The quantity that links the two systems is the "equation of the origins", $\alpha_{\mathrm{CIO}} - \alpha_\Upsilon$ (difference between right ascensions, α_{CIO} and α_Υ, referred to the CIO and

the equinox, respectively), or equivalently ERA−GST. This quantity can be obtained using the SOFA routine E006A. For both transformations, the procedure is to form the various components of expression (5.1), choosing for the $Q(t)$ and $R(t)$ pair either the CIO based or classical forms, and then to combine these components into the complete terrestrial-to-celestial matrix.

In all cases, the polar motion matrix, $W(t)$ in expression (5.1), is needed, using the polar coordinates x_p,y_p. This can be accomplished by calling the SOFA routine POM00 and then transposing the result (*e.g.* using the support routine TR). Also required is the quantity s', modeled by the routine SP00.

SOFA routines that support the IAU 2006/2000A models include the following (among others):

Routine	Brief Description
All-inclusive	
C2I06A	CIO: celestial-to-intermediate matrix, IAU 2006/2000A (Q^{-1})
C2T06A	CIO: celestial-to-terrestrial matrix, IAU 2006/2000A
PNM06A	Equinox: celestial-to-true matrix, IAU 2006/2000A (NPB)
Individual	**Routines delivering a part of the transformation, IAU compliant**
ERA00	Earth Rotation Angle, IAU 2000
GST06A	Greenwich (apparent) Sidereal Time, IAU 2006/2000A
NUT06A	nutation components, IAU 2006/2000A
PFW06	4-angle Fukushima-Williams precession angles, IAU 2006
PN06	Bias, precession, nutation matrices given $\Delta\psi$, $\Delta\epsilon$, IAU 2006/2000A (B, P, N, NPB)
POM00	polar motion matrix
S06	CIO locator s, given X and Y, IAU 2006
S06A	CIO locator s, IAU 2006/2000A
SP00	TIO locator s', IAU 2000
XY06	X, Y, from semi-analytical series, IAU 2006/2000A
XYS06A	X, Y, s, IAU 2006/2000A
Utilities	**Support Routines**
C2IXYS	CIO: celestial-to-intermediate matrix, given X, Y and s (Q^{-1})
C2TCIO	CIO: celestial-to-terrestrial matrix
C2TEQX	Equinox: celestial-to-terrestrial matrix
FW2M	Fukushima-Williams angles to rotation matrix (B, PB or NPB)
TR	Transpose matrix, *e.g.* given Q^{-1} form Q, the terrestrial-to-celestial matrix

The list above is by no means an exhaustive one. Nor is it the only solution to transform between ITRS and GCRS. The matrix for the combined effects of nutation, precession and frame bias is $Q(t)$ in expression (5.1). For the CIO based transformation, this is the intermediate-to-celestial matrix, it can be obtained (as the transpose) using the SOFA routine C2IXYS, starting from the CIP position X,Y and the quantity s that defines the position of the CIO. The IAU 2006/2000A X,Y,s

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[7]The SOFA software collection may be found at the website `http://www.iausofa.org`.

are available by calling the SOFA routine `XYS06A`. In the case of the equinox based transformation, the counterpart to matrix $Q(t)$ is the true-to-celestial matrix. To obtain this matrix requires the nutation components $\Delta\psi$ and $\Delta\epsilon$; these can be predicted using the IAU 2000A model, with adjustments to match IAU 2006 precession, by means of the SOFA routine `NUT06A`. Faster, but less accurate, predictions are available from the `NUT00B` routine, which implements the IAU 2000B truncated model. Once $\Delta\psi$ and $\Delta\epsilon$ are known, the true-to-celestial matrix can be obtained by calling the routine `PN06` and taking the transpose with `TR`.

The intermediate component is the angle for Earth rotation that defines matrix $R(t)$ in expression (5.1). For the CIO based transformation, the angle in question is the Earth Rotation Angle, ERA, which can be obtained by calling the SOFA routine `ERA00`. The counterpart in the case of the equinox based transformation is the Greenwich (apparent) Sidereal Time. This can be obtained by calling the SOFA routine `GST06`, given the celestial-to-true matrix that was obtained earlier.

The three components – the precession-nutation matrix, the Earth rotation quantity and the polar motion matrix – are then assembled into the final terrestrial-to-celestial matrix by means of the SOFA routine `C2TCIO` (CIO based) or `C2TEQX` (equinox based), followed by `TR` as required.

Two methods to generate the terrestrial-to-celestial (*i.e.* ITRS-to-GCRS) matrix $Q(t)R(t)W(t)$, given TT and UT1, are set out below. In each case it is assumed that observed small corrections to the IAU 2006/2000A model, either as $\Delta X, \Delta Y$ or as $d\Delta\psi, d\Delta\epsilon$, are available and need to be included.

Method (1): the CIO based transformation

The CIO based transformation is a function of the CIP coordinates X, Y and the quantity s.

For the given TT, call the SOFA routine `XY06` to obtain the IAU 2006/2000A X, Y from series (see Section 5.5.4) and then the routine `S06` to obtain s. Any CIP corrections $\Delta X, \Delta Y$ can now be applied, and the corrected X, Y, s can be used to call the routine `C2IXYS`, giving the GCRS-to-CIRS matrix. Next call the routine `ERA00` to obtain the ERA corresponding to the current UT1, and apply it as an R_3 rotation using the routine `RZ`, to form the CIRS-to-TIRS matrix. Given x_p, y_p, and obtaining s' by calling the routine `SP00`, the polar motion matrix (*i.e.* TIRS-to-ITRS) is then produced by the routine `POM00`. The product of the two matrices (GCRS-to-TIRS and TIRS-to-ITRS), obtained by calling the routine `RXR`, is the GCRS-to-ITRS matrix, which can be inverted by calling the routine `TR` to give the final result.

Method (2): the equinox based transformation

The classical transformation, based on angles and using sidereal time is also available.

Given TT, the IAU 2006/2000A nutation components $\Delta\psi, \Delta\epsilon$ are obtained by calling the SOFA routine `NUT06A`. Any corrections $d\Delta\psi, d\Delta\epsilon$ can now be applied. Next, the GCRS-to-true matrix is obtained using the routine `PN06` (which employs the 4-rotation Fukushima-Williams method described in Section 5.3.4, final paragraph). The classical GCRS-to-true matrix can also be generated by combining separate frame bias, precession and nutation matrices. The SOFA routine `BI00` can be called to obtain the frame bias components, the routine `P06E` to obtain various precession angles, and the routine `NUM06A` to generate the nutation matrix. The product N×P×B is formed by using the routine `RXR`. Next call the routine `GST06` to obtain the GST corresponding to the current UT1, and apply it as an R_3 rotation using the routine `RZ` to form the true matrix-to-TIRS. Given x_p, y_p, and obtaining s' with the routine `SP00`, the polar motion matrix (*i.e.* TIRS-to-ITRS) is then obtained using the routine `POM00`. The product of the two matrices (GCRS-to-TIRS and TIRS-to-ITRS), obtained by calling the routine `RXR`, is the GCRS-to-ITRS matrix, which can be inverted by calling the routine `TR` to give the final result.

Methods (1) and (2) agree to microarcsecond precision.

Both methods can be abridged to trade off speed and accuracy (see Capitaine and Wallace, 2008). The abridged nutation model IAU 2000B (see Section 5.5.1) can be substituted in Method (2) by calling `NUT00B` instead of `NUT06A`. Depending on the application, the best compromise between speed and accuracy may be to evaluate the full series to obtain sample values for interpolation.

5.10 Notes on the new procedure to transform from ICRS to ITRS

The transformation between the GCRS and the ITRS, which is provided in detail in this chapter for use in the IERS Conventions, is also part of the more general transformation for computing directions of celestial objects in intermediate systems or terrestrial systems.

The procedure to be followed in transforming from the celestial (ICRS) to the terrestrial (ITRS) systems has been clarified to be consistent with the improving observational accuracy. See Figure 5.1 in the IAU NFA Working Group documents (Capitaine *et al.*, 2007) for a diagram of the CIO based and equinox based procedures to be followed.

The purpose of this chart is to show the ICRS-to-BCRS-to-GCRS-to-ITRS transformation in general relativity (IAU 2000 Resolution B1.3) and the parallel CIO and equinox based processes (IAU 2000 Resolution B1.8).

As before, we make use of celestial and terrestrial intermediate reference systems in transforming to a terrestrial reference system (See also Seidelmann and Kovalevsky (2002).)

The Celestial Intermediate Pole (CIP) that is realized by the IAU 2006/2000A precession-nutation model defines its equator and the Conventional Intermediate Origin replaces the equinox.

The position in this reference system is called the intermediate right ascension and declination and is analogous to the previous designation of "apparent right ascension and declination."
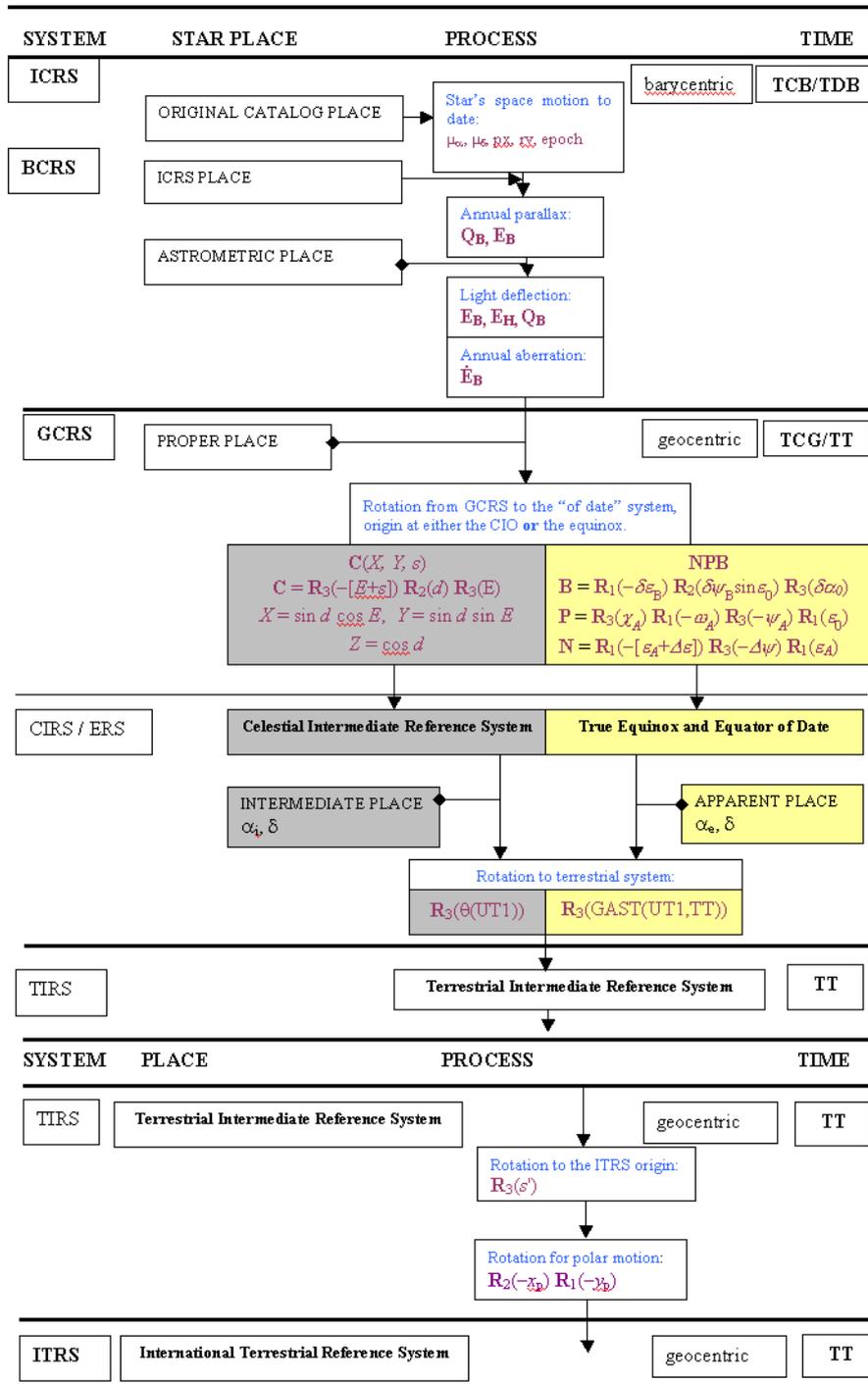
SYSTEM **STAR PLACE** **PROCESS** **TIME**

ICRS

ORIGINAL CATALOG PLACE

Star's space motion to date:
μ_α, μ_δ, px, rv, epoch

barycentric TCB/TDB

BCRS

ICRS PLACE

Annual parallax:
$\mathbf{Q_B}$, $\mathbf{E_B}$

ASTROMETRIC PLACE

Light deflection:
$\mathbf{E_B}$, $\mathbf{E_H}$, $\mathbf{Q_B}$

Annual aberration:
$\dot{\mathbf{E}}_\mathbf{B}$

GCRS

PROPER PLACE

geocentric TCG/TT

Rotation from GCRS to the "of date" system, origin at either the CIO **or** the equinox.

$C(X, Y, s)$
$\mathbf{C} = \mathbf{R_3}(-[E+s])\,\mathbf{R_2}(d)\,\mathbf{R_3}(E)$
$X = \sin d \cos E, \quad Y = \sin d \sin E$
$Z = \cos d$

NPB
$\mathbf{B} = \mathbf{R_1}(-\delta\varepsilon_B)\,\mathbf{R_2}(\delta\psi_B\sin\varepsilon_0)\,\mathbf{R_3}(\delta\alpha_0)$
$\mathbf{P} = \mathbf{R_3}(\chi_A)\,\mathbf{R_1}(-\omega_A)\,\mathbf{R_3}(-\psi_A)\,\mathbf{R_1}(\varepsilon_0)$
$\mathbf{N} = \mathbf{R_1}(-[\varepsilon_A+\Delta\varepsilon])\,\mathbf{R_3}(-\Delta\psi)\,\mathbf{R_1}(\varepsilon_A)$

CIRS / ERS

Celestial Intermediate Reference System | **True Equinox and Equator of Date**

INTERMEDIATE PLACE
α_i, δ

APPARENT PLACE
α_e, δ

Rotation to terrestrial system:
$\mathbf{R_3}(\theta(UT1))$ | $\mathbf{R_3}(GAST(UT1,TT))$

TIRS **Terrestrial Intermediate Reference System** TT

SYSTEM **PLACE** **PROCESS** **TIME**

TIRS **Terrestrial Intermediate Reference System** geocentric TT

Rotation to the ITRS origin:
$\mathbf{R_3}(s')$

Rotation for polar motion:
$\mathbf{R_2}(-x_p)\,\mathbf{R_1}(-y_p)$

ITRS **International Terrestrial Reference System** geocentric TT

Figure 5.1: Process to transform from celestial to terrestrial reference systems (Chart from the IAU Working Group on Nomenclature for Fundamental Astronomy (2006)). The chart summarizes the system, and the elements that are associated with that system, *i.e.* the name for the positions (place), the processes/corrections, the origin to which the coordinates are referred, and the time scale to use. In particular the blue type in the box in the "Process" column is the operation/correction to be applied, and the purple type indicates the quantities required for that process. CIO and equinox based processes are indicated using grey and yellow shading, respectively.

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